5-1 and 5-2 Objectives

- To use properties of midsegments to solve problems
- To use properties of perpendicular bisectors and angle bisectors

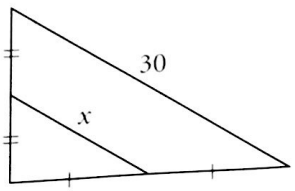
A **midsegment** of a triangle is a segment that connects the midpoints of two sides. A midsegment is parallel to the third side, and is half its length.

In a **coordinate proof,** a figure is drawn on a coordinate plane and formulas are used to prove properties of the figure.

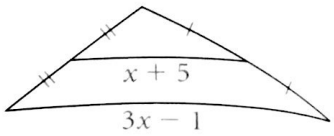
The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. The Perpendicular Bisector Theorem together with its converse states that a point is on the perpendicular bisector of a segment if and only if it is equidistant from the endpoints of the segment. The Angle Bisector Theorem together with its converse states that a point is on the bisector of an angle if and only if it is equidistant from the sides of the angle.

x^2 Algebra Find the value of x.

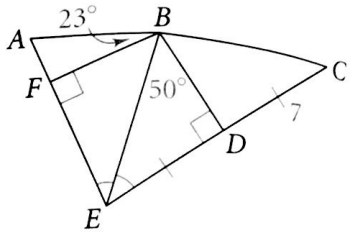
11.



12.



Use the figure to find each segment length or angle measure.



13. $m\angle BEF$ **14.** FE

15. EC **16.** $m\angle CEA$

5-3 Objectives

- To identify properties of perpendicular bisectors and angle bisectors
- To identify properties of medians and altitudes of a triangle

When three or more lines intersect in one point, they are **concurrent**.

The **median of a triangle** is a segment whose endpoints are a vertex and the midpoint of the opposite side. The **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side.

For any given triangle, special segments and lines are concurrent:

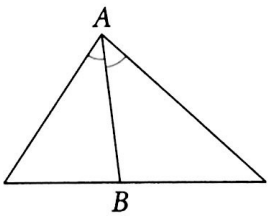
- the perpendicular bisectors of the sides at the circumcenter, the center of the circle that can be **circumscribed about** the triangle
- the bisectors of the angles at the incenter, the center of the circle that can be **inscribed in** the triangle
- the medians at the **centroid**
- the lines containing the altitudes at the **orthocenter of the triangle.**

Graph $\triangle ABC$ with vertices $A(2, 3)$, $B(-4, -3)$, and $C(2, -3)$. Find the coordinates of each point of concurrency.

17. circumcenter **18.** centroid **19.** orthocenter

Determine whether $\overline{AB}$ is a perpendicular bisector, an angle bisector, a median, an altitude, or none of these. Explain.

20.



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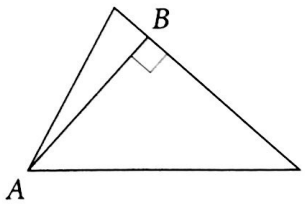

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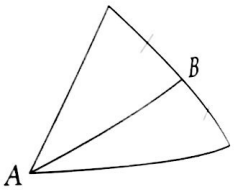


5-4 Objectives

- To write the negation of a statement and the inverse and contrapositive of a conditional statement
- To use indirect reasoning

The **negation** of a statement has the opposite truth value. The **inverse** of a conditional statement is the negation of both the hypothesis and the conclusion. The **contrapositive** of a conditional statement switches the hypothesis and the conclusion and negates both. Statements that always have the same truth value are **equivalent statements.**

To use **indirect reasoning,** consider all possibilities and then prove all but one false. The remaining possibility must be true.

The three steps of an **indirect proof** are:

Step 1 State as an assumption the opposite (negation) of what you want to prove.

Step 2 Show that this assumption leads to a contradiction.

Step 3 Conclude that the assumption must be false and that what you want to prove must be true.

Write the inverse and the contrapositive of each statement.

23. If it is snowing, then it is cold outside.

24. If an angle is obtuse, then its measure is greater than 90 and less than 180.

25. If a figure is a square, then its sides are congruent.

26. If you are in Australia, then you are south of the equator.

Write a convincing argument that uses indirect reasoning.

27. The product of two numbers is even. Show that at least one of the two numbers must be even.

28. Show that a right angle cannot be formed by the intersection of nonperpendicular lines.

29. Show that a triangle can have at most one obtuse angle.

30. Show that an equilateral triangle cannot have an obtuse angle.

5-5 Objectives

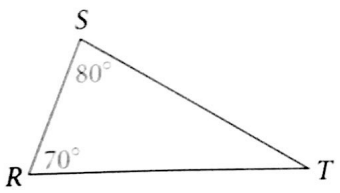
- To use inequalities involving angles of triangles
- To use inequalities involving sides of triangles

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side. The converse is also true. If two angles are not congruent, then the longer side lies opposite the larger angle.

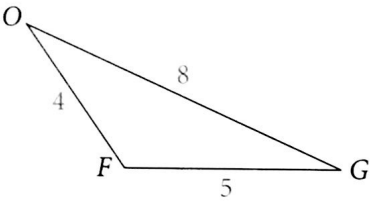
The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

List the angles and sides in order from smallest to largest.

31.



32.



Is it possible for a triangle to have sides with the given lengths? Explain.

33. 5 in., 8 in., 15 in.

34. 10 cm, 12 cm, 20 cm

35. 20 m, 22 m, 24 m

36. 3 ft, 6 ft, 8 ft

37. 1 yd, 1 yd, 3 yd

38. 5 km, 6 km, 7 km

Two side lengths of a triangle are given. Write an inequality to show the range of values, x, for the length of the third side.

39. 4 in., 7 in.

40. 8 m, 15 m

41. 2 cm, 8 cm

42. 12 ft, 13 ft