Using Graphs to relate Two Quantities

Section 5-1

Using Graphs

- Graphs can be used to visually represent the relationship between two variable quantities as they each change.
- A graph is the most understandable way of showing how one variable changes with respect to another variable.
- Graphs can show changes in speed, altitude, distance, volume, time, and other variable quantities.

Graphing Relationships

State whether each word or phrase represents an amount that is increasing, decreasing, or constant.

1. stays the same constant

2. rises increasing

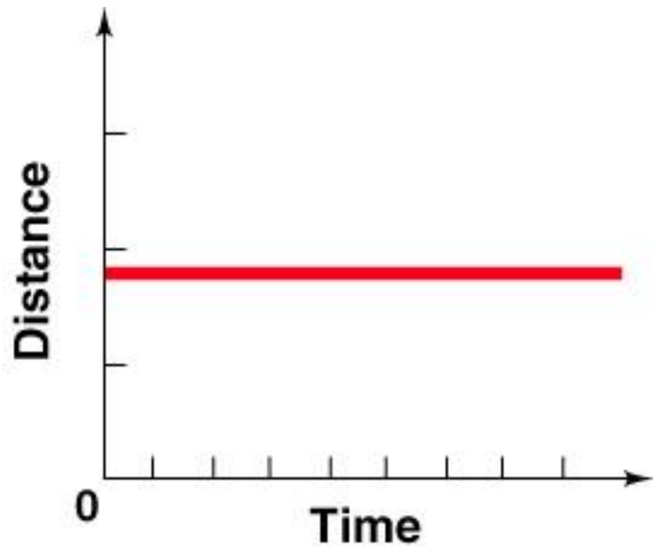
3. drops decreasing

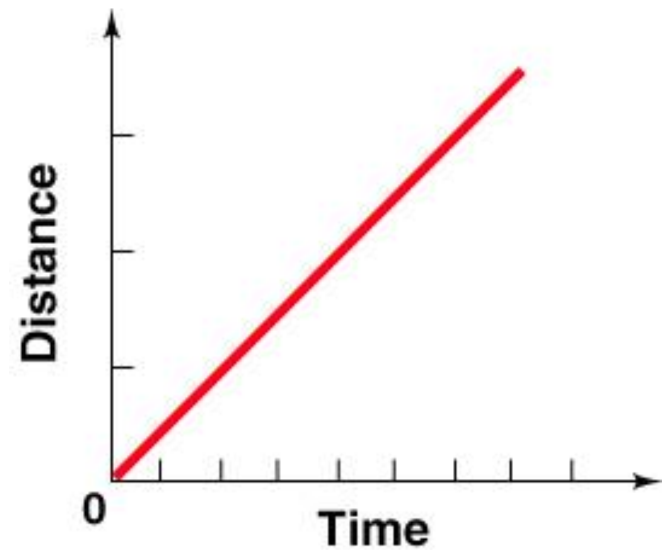
4. slows down decreasing

Example: Using Graphs

Which graph could show a car sitting at a stoplight?

How do you know?





Graph I indicates a quantity that does not change with time.

Graph II shows an increase over time.

A car sitting at a stoplight would stay in the same place over time.

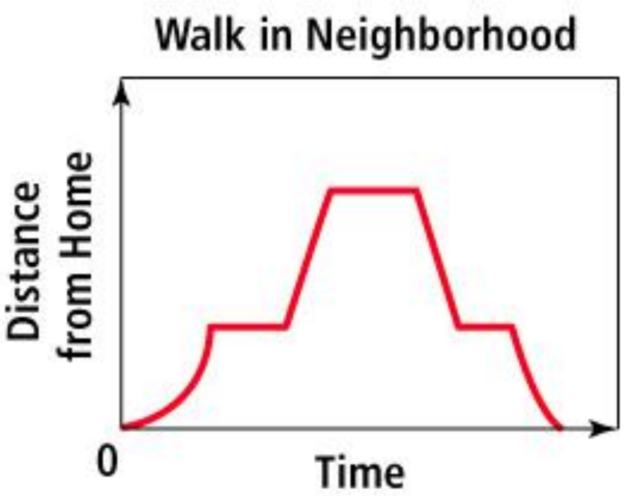
Graph I could show a car sitting at a stoplight.

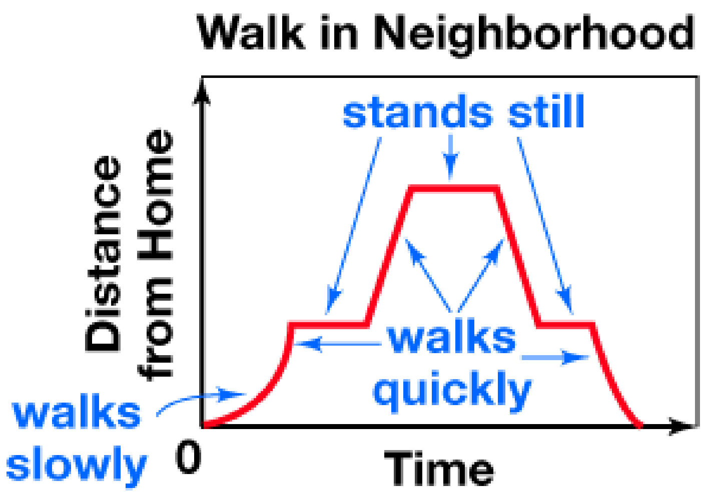
Example: Using Graphs

A pelican flies above the water searching for fish. Sketch a graph of its altitude from takeoff from shore to diving to the water to catch a fish. Label each section.

Your Turn:

This graph shows someone taking a walk in the neighborhood. Describe what it shows by labeling each part.

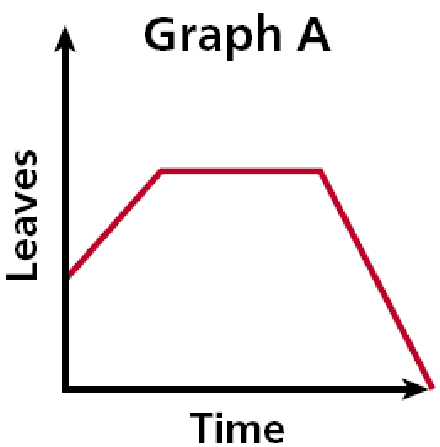


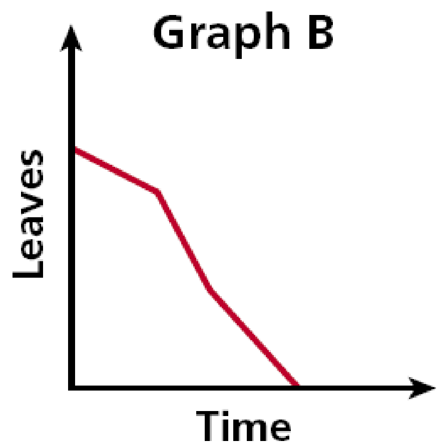


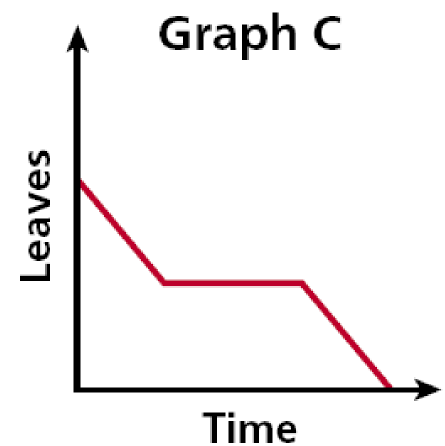
Example:

To relate a graph to a given situation, use key words in the description.

Each day several leaves fall from a tree. One day a gust of wind blows off many leaves. Eventually, there are no more leaves on the tree. Choose the graph that best represents the situation.



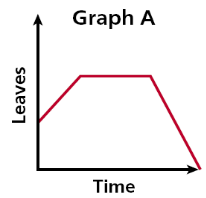


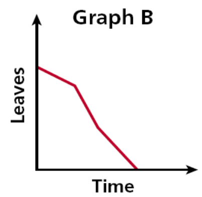


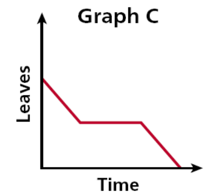
Step 1 Read the graphs from left to right to show time passing.

Example: Continued

Step 2 List key words in order and decide which graph shows them.





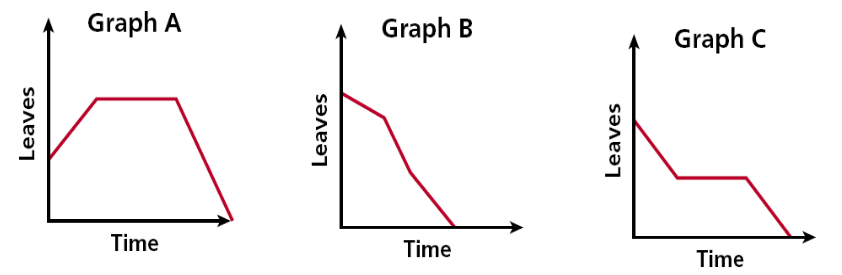


Key Words	Segment Description	Graphs…
Each day several leaves fall	Never horizontal	Graph B
Wind blows off many leaves	Slanting downward rapidly	Graphs A, B, and C
Eventually no more leaves	Slanting downward until reaches zero	Graphs A, B, and C

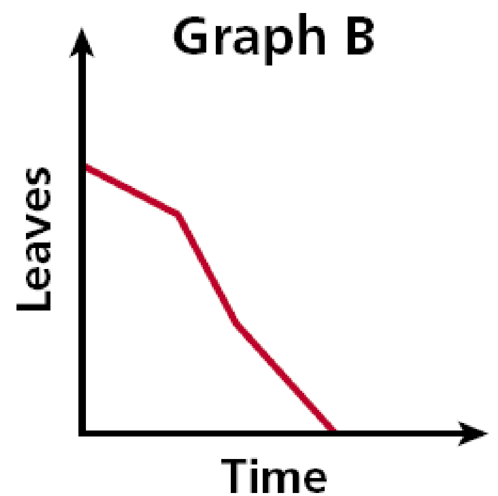
Example: Continued

Step 3 Pick the graph that shows all the key phrases in order.

Never horizontal, slanting downward rapidly, slanting downward until reaching zero.

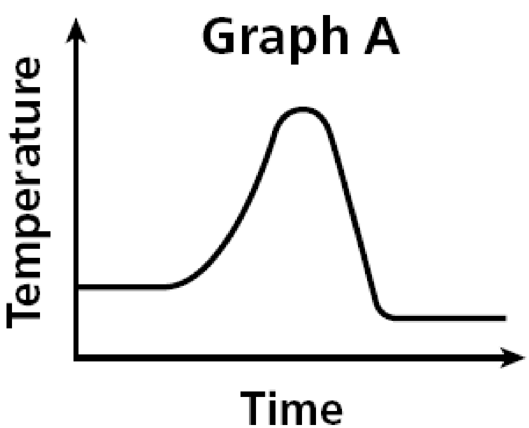


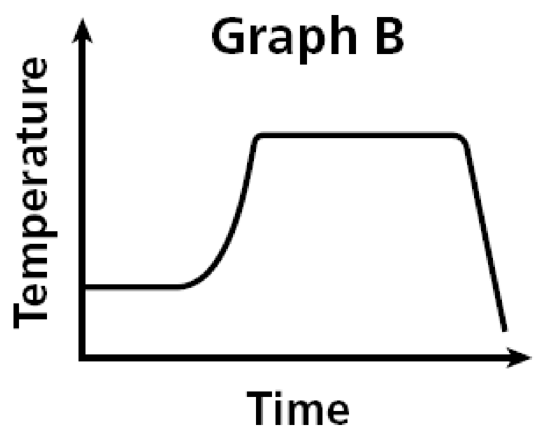
The correct graph is B.

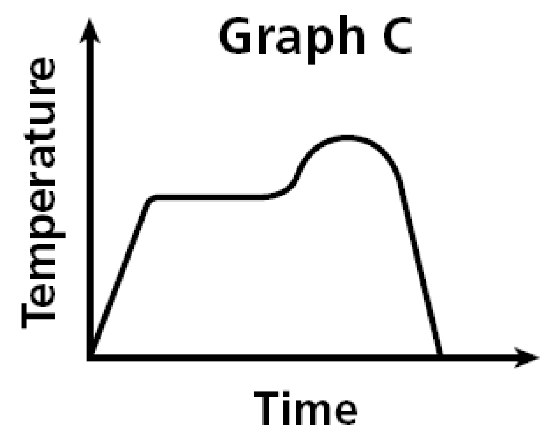


Your Turn:

The air temperature increased steadily for several hours and then remained constant. At the end of the day, the temperature increased slightly before dropping sharply. Choose the graph that best represents this situation.



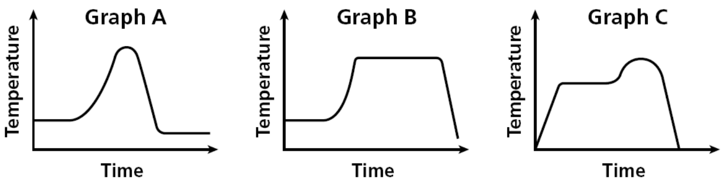




Step 1 Read the graphs from left to right to show time passing .

Your Turn: Continued

Step 2 List key words in order and decide which graph shows them.

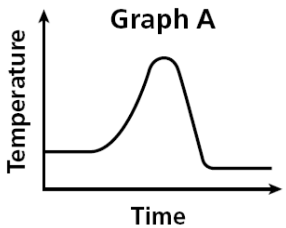


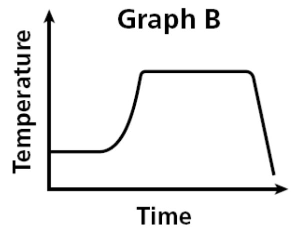
Key Words	Segment Description	Graphs…
Increased steadily	Slanting upward	Graph C
Remained constant	Horizontal	Graphs A, B, and C
Increased slightly before dropping sharply	Slanting upward and then steeply downward	Graph C

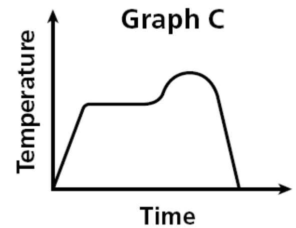
Your Turn: Continued

Step 3 Pick the graph that shows all the key phrases in order.

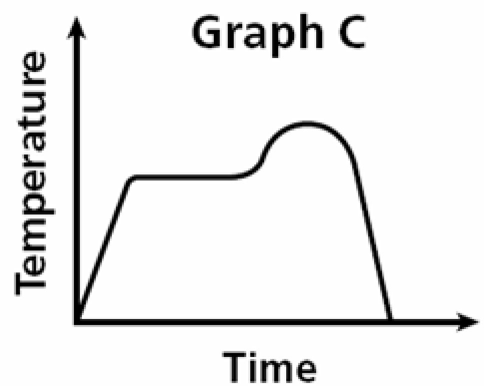
Slanting upward, horizontal, slanting upward and then steeply downward







The correct graph is graph C.



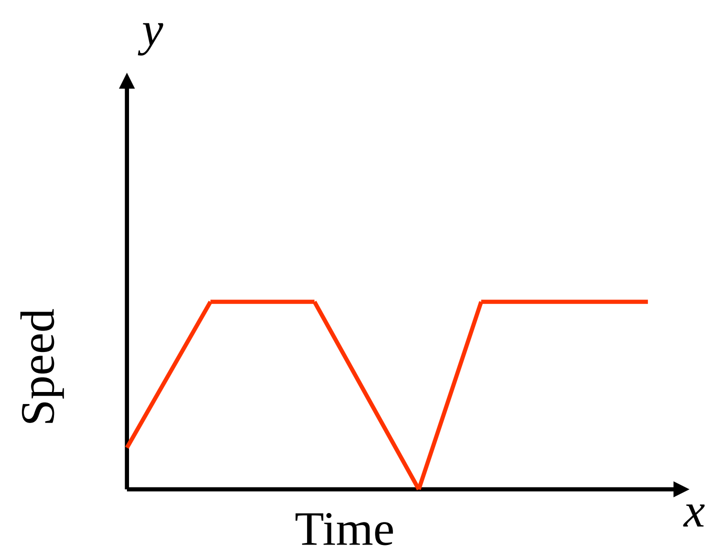
Example: Sketching Graphs

Sketch a graph for the situation.

A truck driver enters a street, drives at a constant speed, stops at a light, and then continues.

As time passes during the trip (moving left to right along the *x*-axis), the truck's speed (*y*-axis) does the following:

- initially increases
- remains constant
- decreases to a stop
- increases
- remains constant



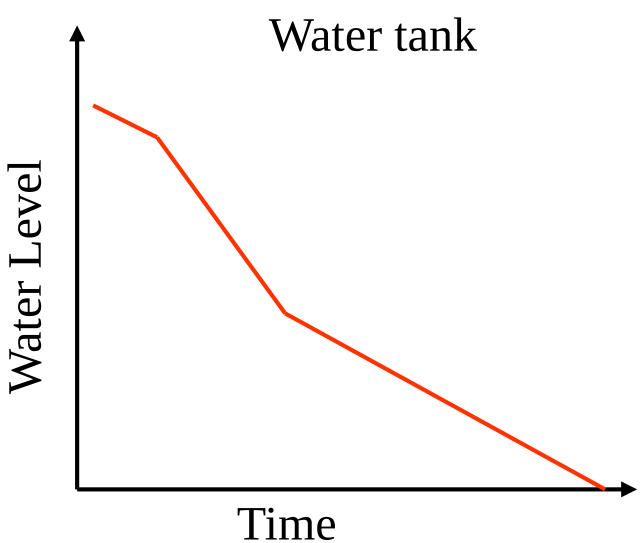
Helpful Hint

When sketching or interpreting a graph, pay close attention to the labels on each axis.

Your Turn:

Sketch a graph for the situation.

Henry begins to drain a water tank by opening a valve. Then he opens another valve. Then he closes the first valve. He leaves the second valve open until the tank is empty.

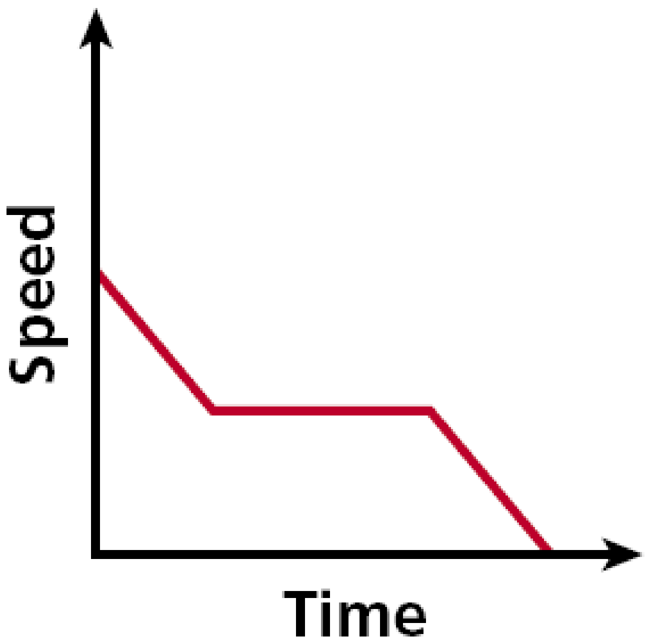


As time passes while draining the tank (moving left to right along the *x*-axis), the water level (*y*-axis) does the following:

- initially declines
- declines more rapidly
- and then the decline slows down.

Example: Writing a Situation for a Graph

Write a possible situation for the given graph.



Step 1 Identify labels. *x*-axis: time
y-axis: speed

Step 2 Analyze sections. over time, the speed:

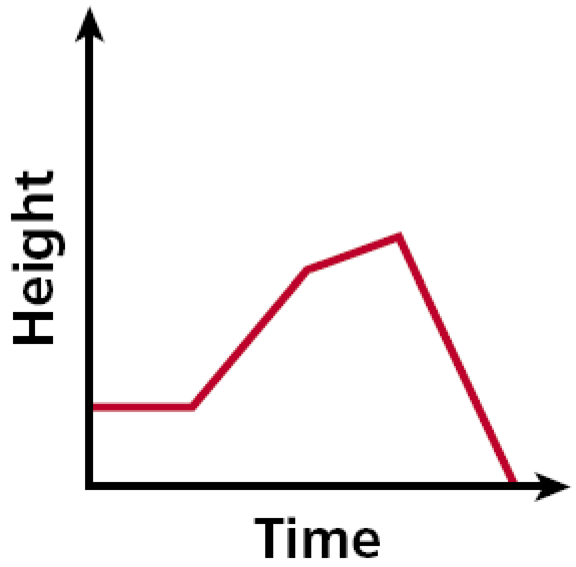
- initially decreases,
- remains constant,
- and then decreases to zero.

Possible Situation:

A car approaching traffic slows down, drives at a constant speed, and then slows down until coming to a complete stop.

Your Turn:

Write a possible situation for the given graph.



Possible answer: The level of water in a bucket stays constant. A steady rain raises the level. The rain slows down. Someone dumps the bucket.

Example: Relating Tables and Graphs

In Algebra, we use multiple representations (verbal description, table, graph, equation) to describe data or a relationship between two variables.

Marcus and Janine made the table shown below to represent the difference between their ages during different years.

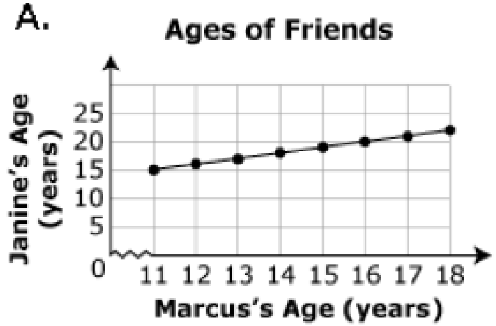
Which graph matches the information in the table?

Ages of Friends in Years

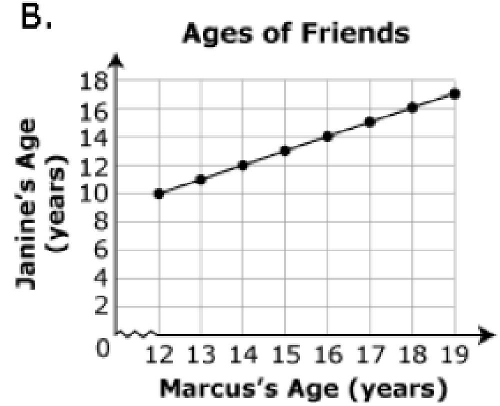
Marcus's Age	12	13	14	15	16	17	18	19
Janine's Age	15	16	17	18	19	20	21	22

Answer: D

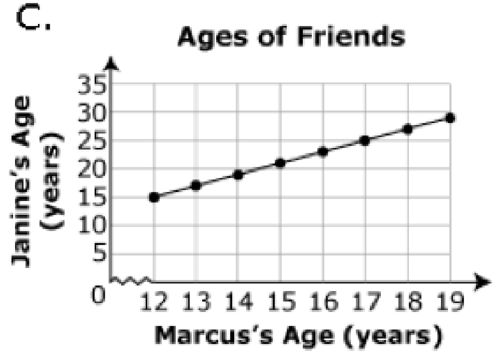
A.



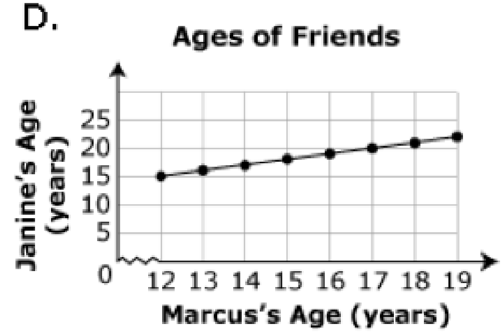
B.



C.



D.



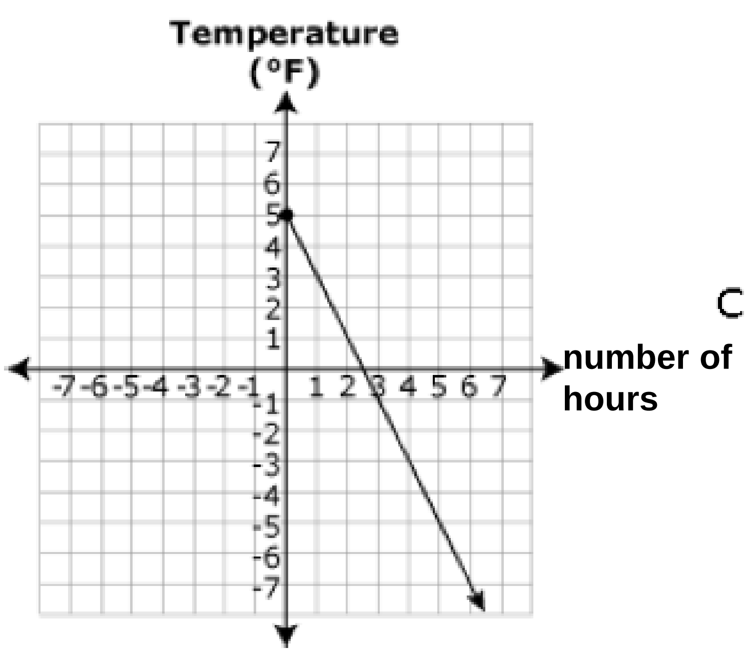
Your turn:

Ethan heard a weather report that stated the temperature in Wichita would drop from 5 degrees Fahrenheit at a rate of 2 degrees every hour.

Which table matches the information in the graph that Ethan made?

Answer: A

Temperature in Wichita



A.

Temperature in Wichita

Number of Hours	Temperature (°F)
0	5
1	3
2	1
3	-1

B.

Temperature in Wichita

Number of Hours	Temperature (°F)
0	2.5
1	2
2	1.5
3	1

C.

Temperature in Wichita

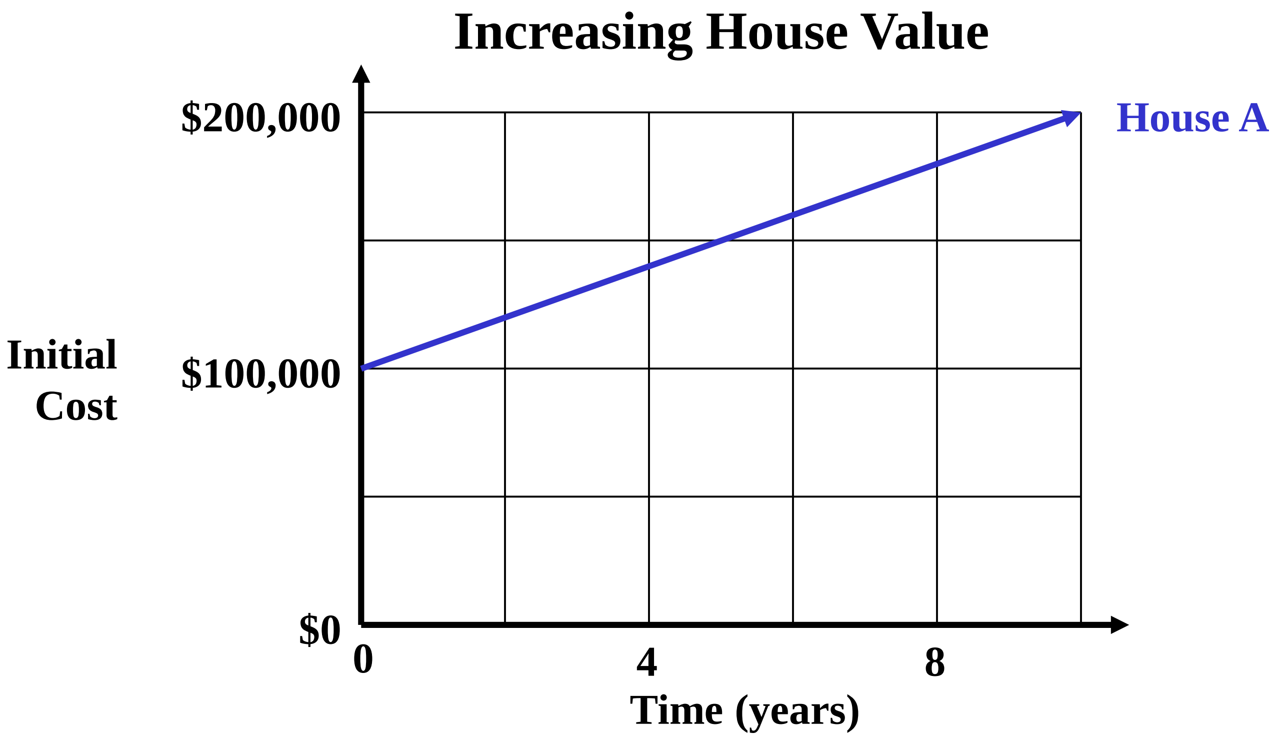
Number of Hours	Temperature (°F)
1	5
2	3
3	1
4	-1

D.

Temperature in Wichita

Number of Hours	Temperature (°F)
5	0
3	1
1	2
-1	-3

Example: Interpreting Graphs and Tables



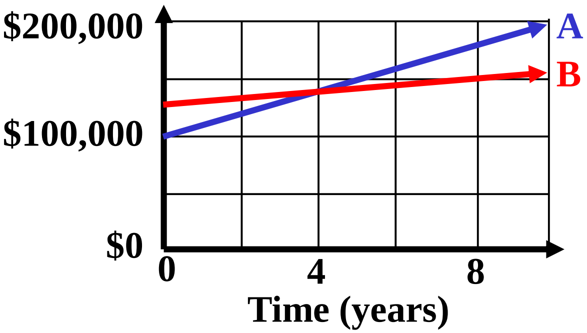
House A cost $100,000 and increased in value as shown in the graph.

House B cost less than house A and increased in value at a greater rate. Which graph on the next slide illustrates this situation.

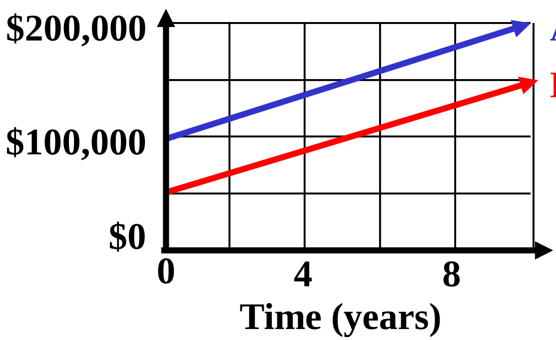
Example: Continued

House A cost $100,000 and increased in value as shown in the graph. House B cost less than house A and increased in value at a greater rate.

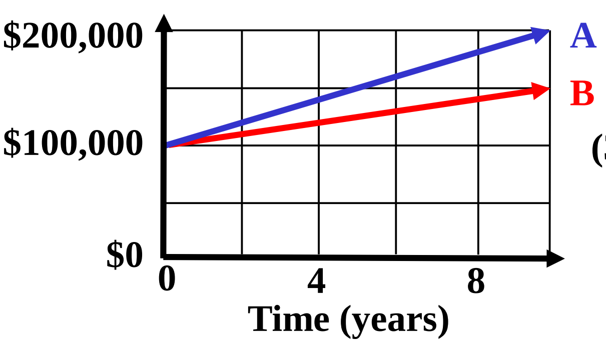
Which One?



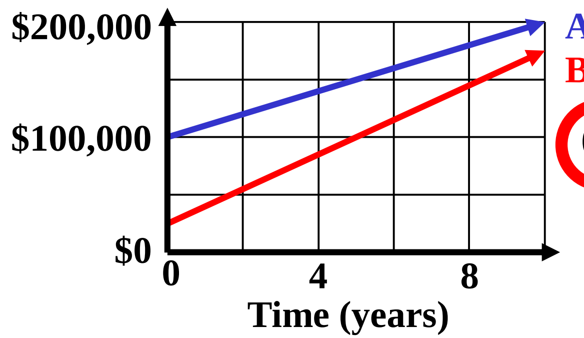
(1)



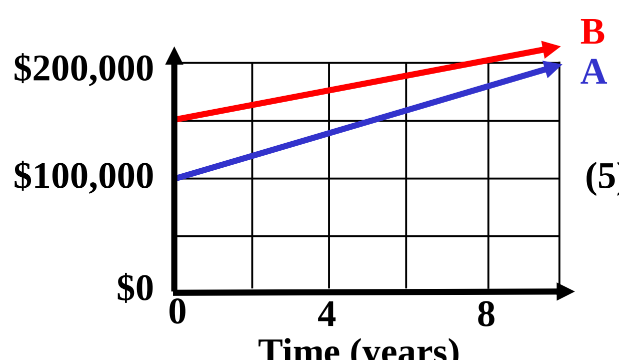
(2)



(3)



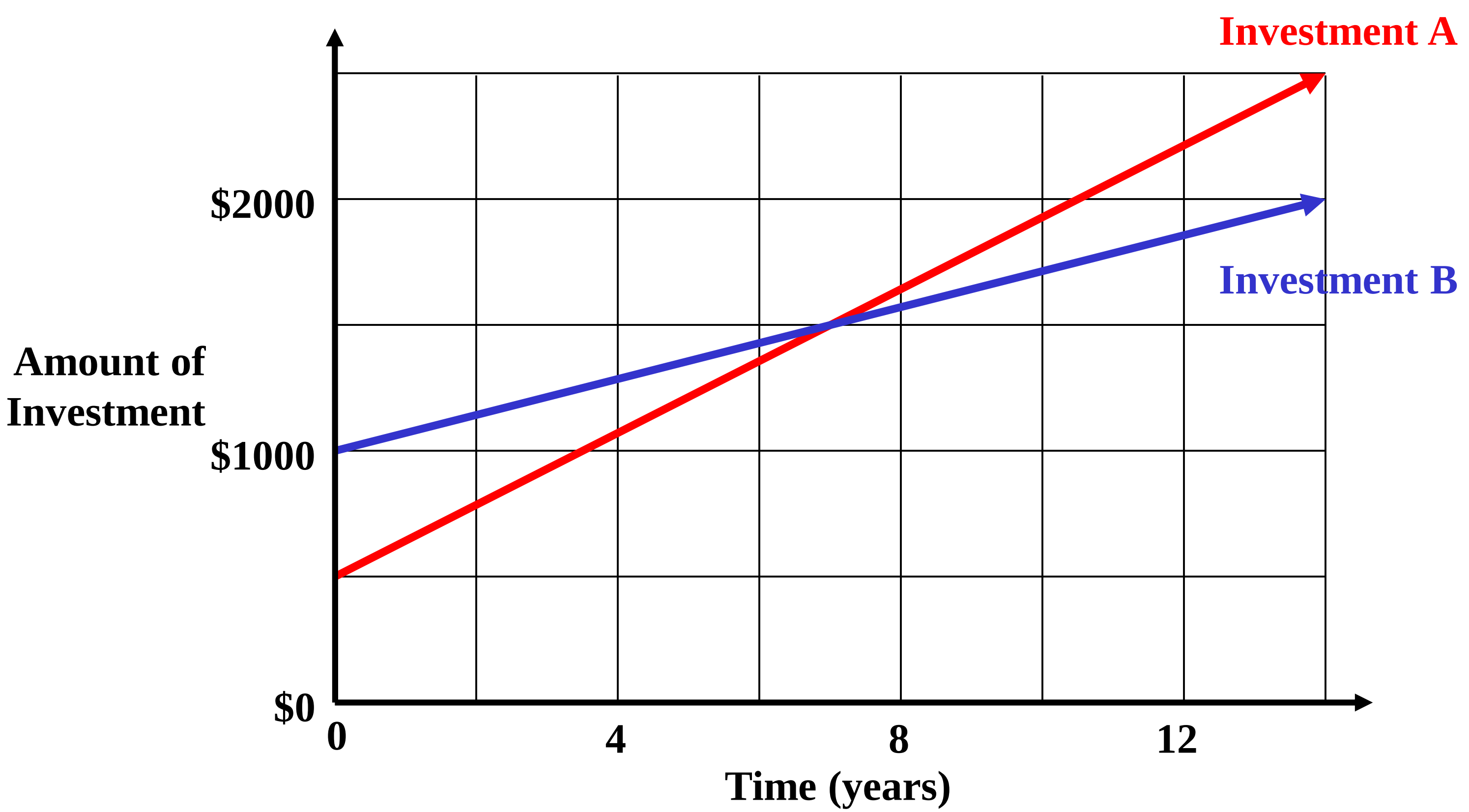
(4)



(5)

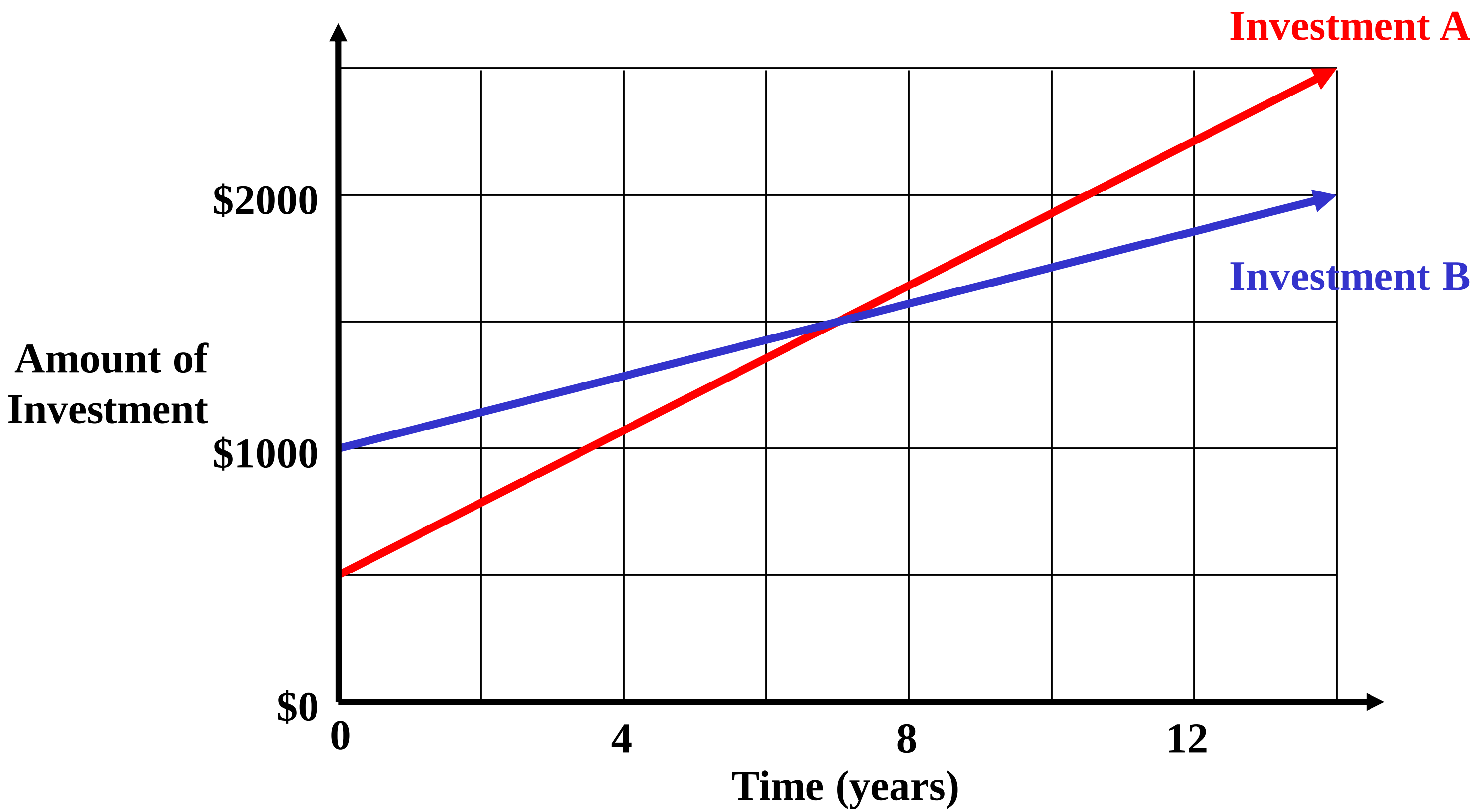
Your Turn:

The changing values of two investments are shown in the graph below.

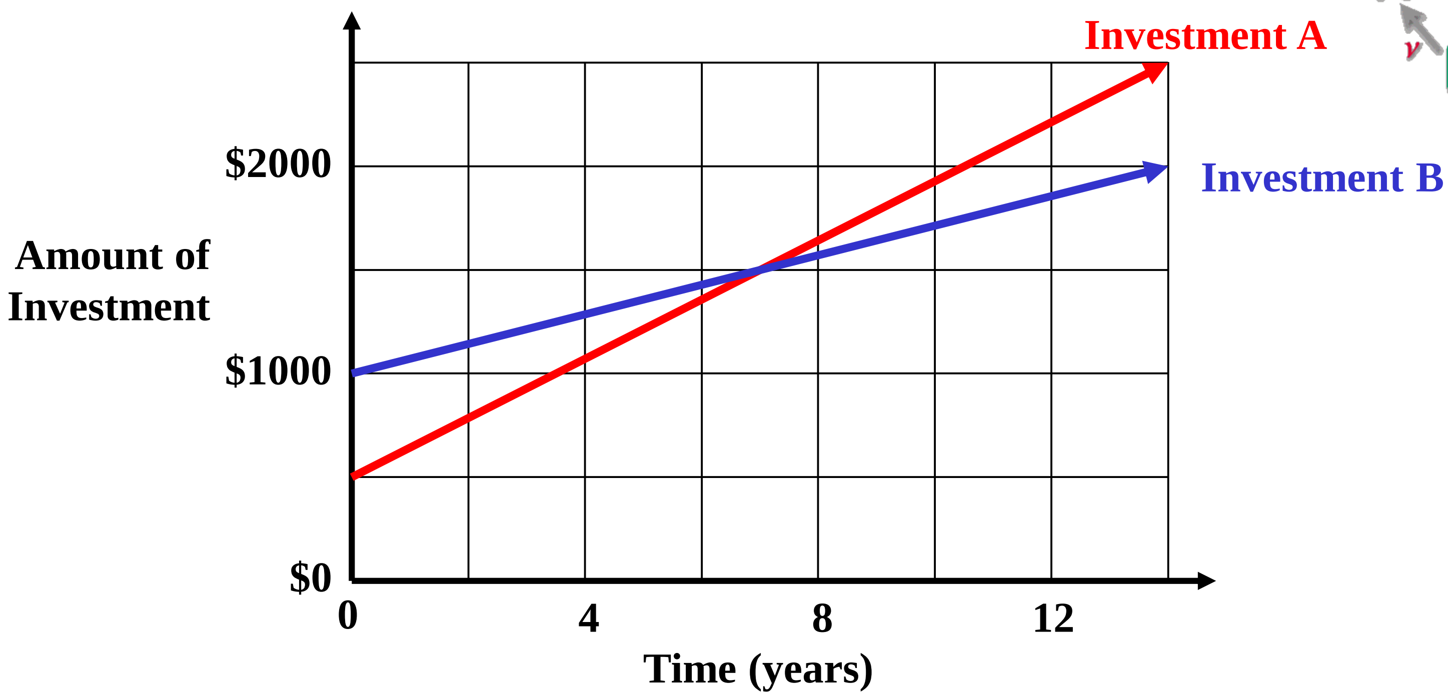


Your Turn: Continued

How does the amount initially invested and the rate of increase for investment A compare with those of investment B?



Your Turn: Continued



Compared to investment B, investment A had a

(1) lesser initial investment and a lesser rate of increase.

(2) lesser initial investment and the same rate of increase.

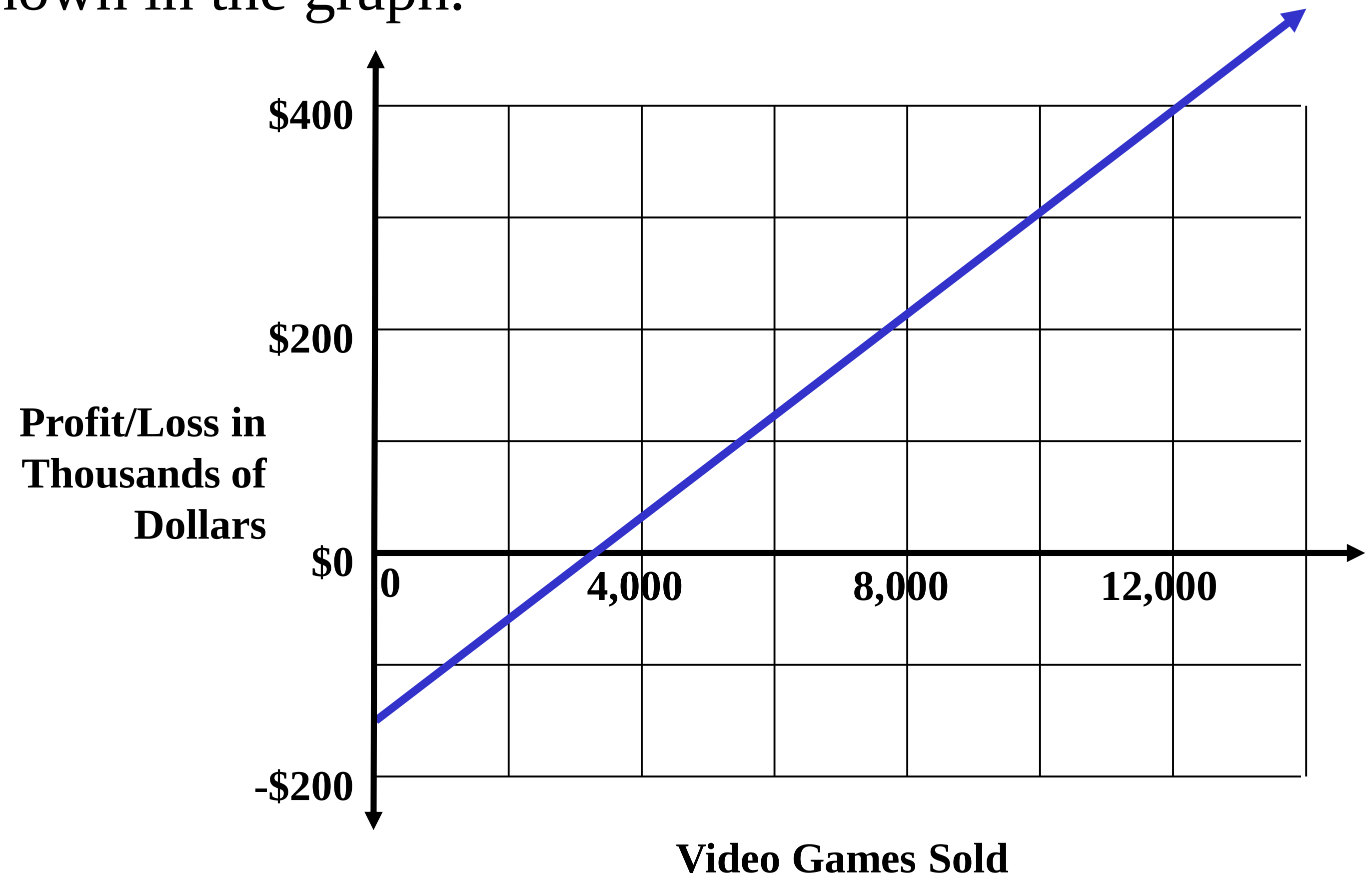
(3) lesser initial investment and a greater rate of increase.

(4) greater initial investment and a lesser rate of increase.

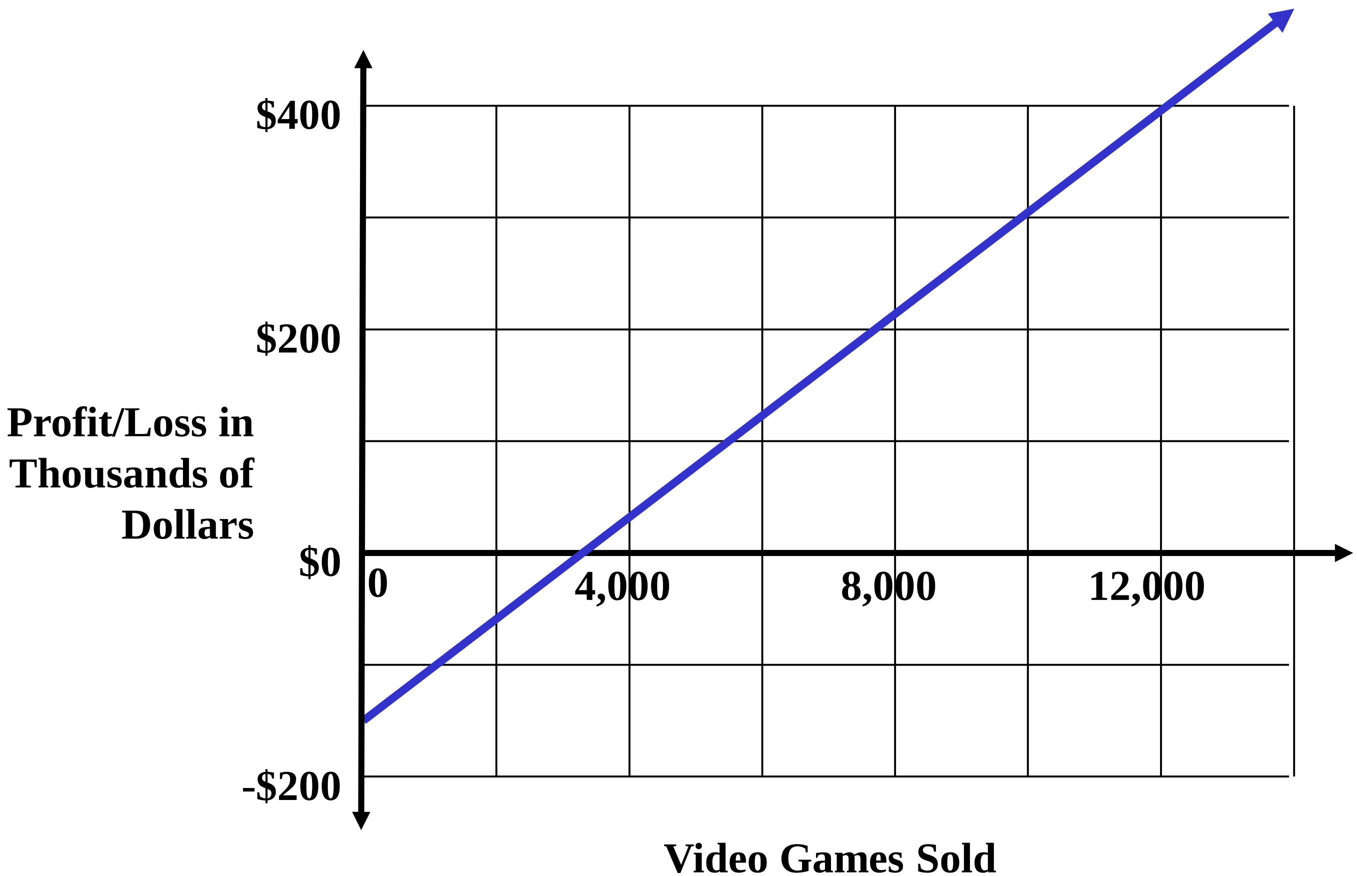
(5) greater initial investment and a greater rate of increase.

Your Turn:

The profit, in thousands of dollars, that a company expects to make from the sale of a new video game is shown in the graph.



Your Turn: Continued

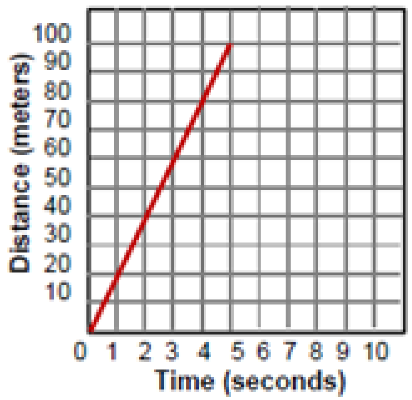


What is the expected profit/loss before any video games are sold?

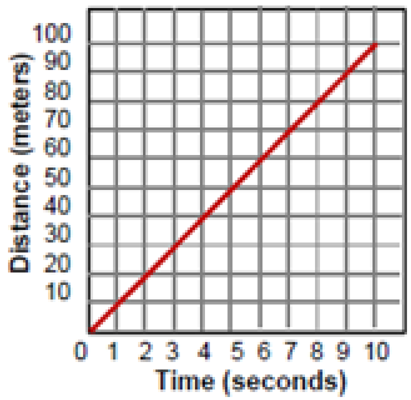
(1) $0 (2) -$150 (3) -$250 (4) -$150,000 (5) -$250,000

Your Turn:

Object A



Object B



Object C

Time (seconds)	Distance (meters)
0	0
3	10
6	20
9	30

Object C moves at constant speed.

Object D

Time (seconds)	Distance (meters)
0	0
1.5	10
3	20
4.5	30

Object D moves at constant speed.

The speed of an object is defined as the change in distance divided by the change in time.

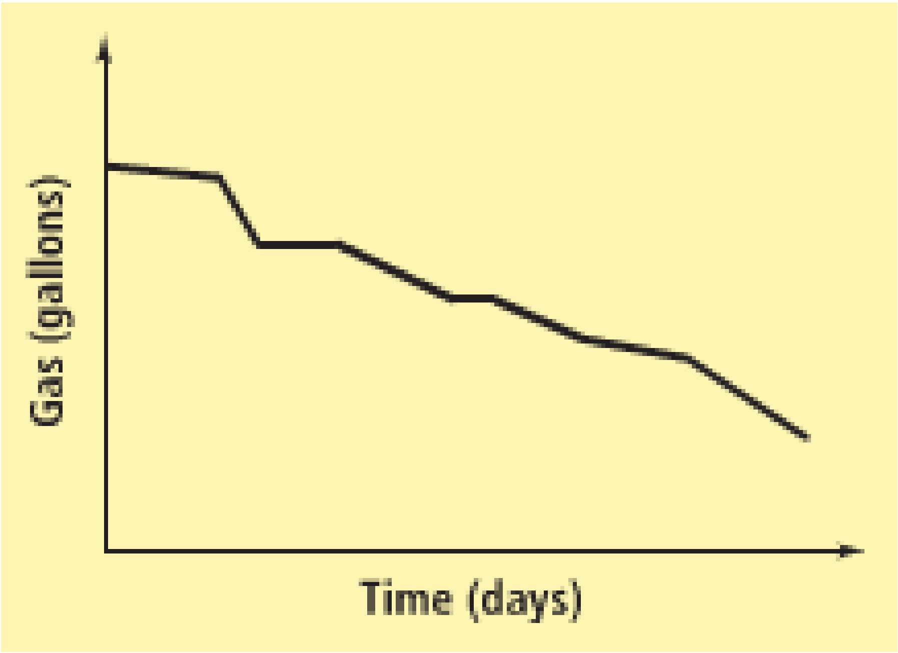
Information about objects A, B, C and D are shown in the graphs and tables.

Based on the information given, drag and drop the object names in order from greatest speed to least speed in the table provided.

Object A	Greatest Speed	A
Object B		B
Object C		D
Object D	Least Speed	C

Your Turn: Analyzing a Graph

The graph below shows the amount of gasoline in Jamie's tank after she fills up her car. (1) What are the variables? (2) Describe how they are related at various points on the graph.



Answers:

(1) The variables are the amount of gas (in gallons) and time (in days).

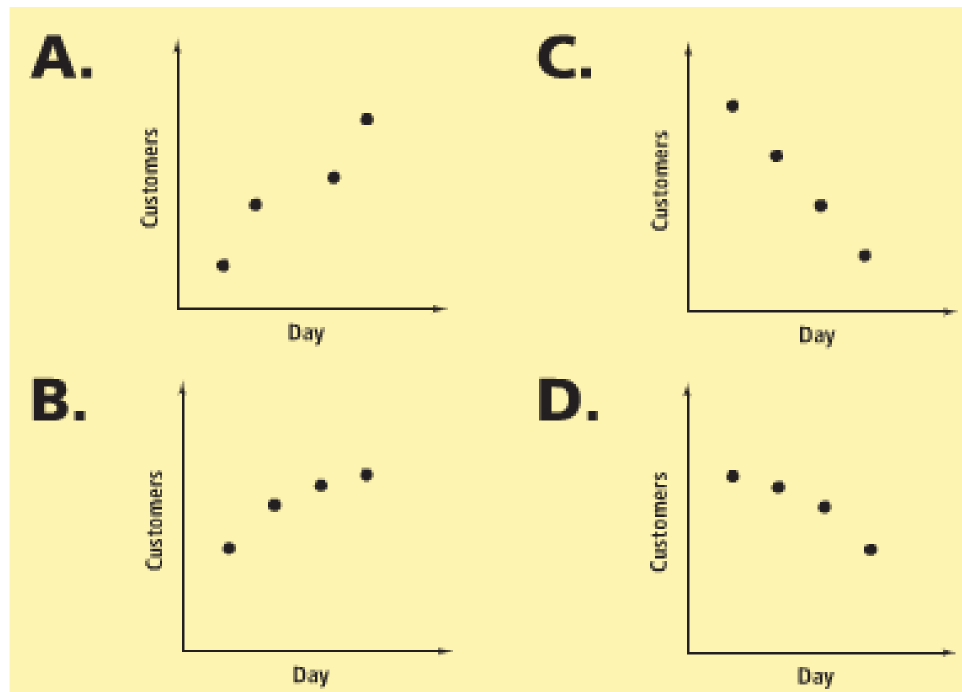
(2) The amount of gas decreases each time Jamie drives somewhere and stays constant when she is not driving.

Your Turn: Matching a Table and a Graph

The table shows the total number of customers at a car wash after 1, 2, 3, and 4 days of its grand opening. Which graph could represent the data shown in the table?

Car Wash Grand Opening

Day	Total Customers
1	61
2	125
3	177
4	242



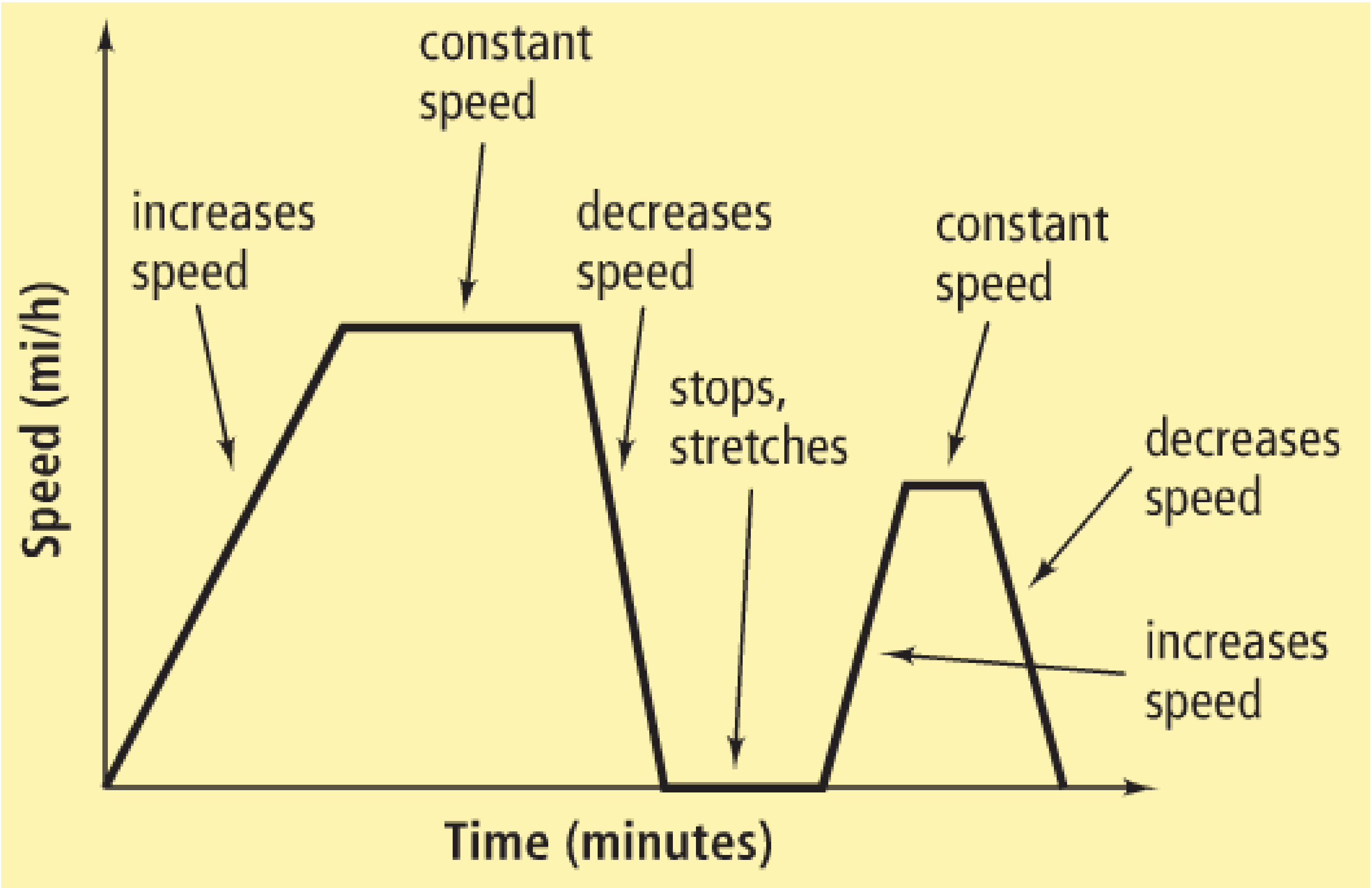
Answer: A

Your Turn: Sketching a Graph

When Malcolm jogs on the treadmill, he gradually increases his speed until he reaches a certain level. Then he jogs at this level for several minutes. Then he slows to a stop and stretches. After this he increases to a speed that is slightly lower than before and jogs at this speed for a short while before slowing to a stop again.

What is a possible sketch of a graph that shows Malcolm's jogging speed during his workout? Label each section.

Your Turn: Answer



Assignment

Pg. 238-239 # 1-21 all

Relations and Functions

Section 5-2

Review

- A **relation** between two variables *x* and *y* is a set of ordered pairs
- An **ordered pair** consist of a *x* and *y*-coordinate
  - **A relation** may be viewed as *ordered pairs*, *mapping design*, *table*, *equation*, *or written in sentences*
- *x*-values are **inputs, domain, independent variable**
- *y*-values are **outputs**, **range, dependent variable**

Example 1

$$\{(0,-5),(1,-4),(2,-3),(3,-2),(4,-1),(5,0)\}$$

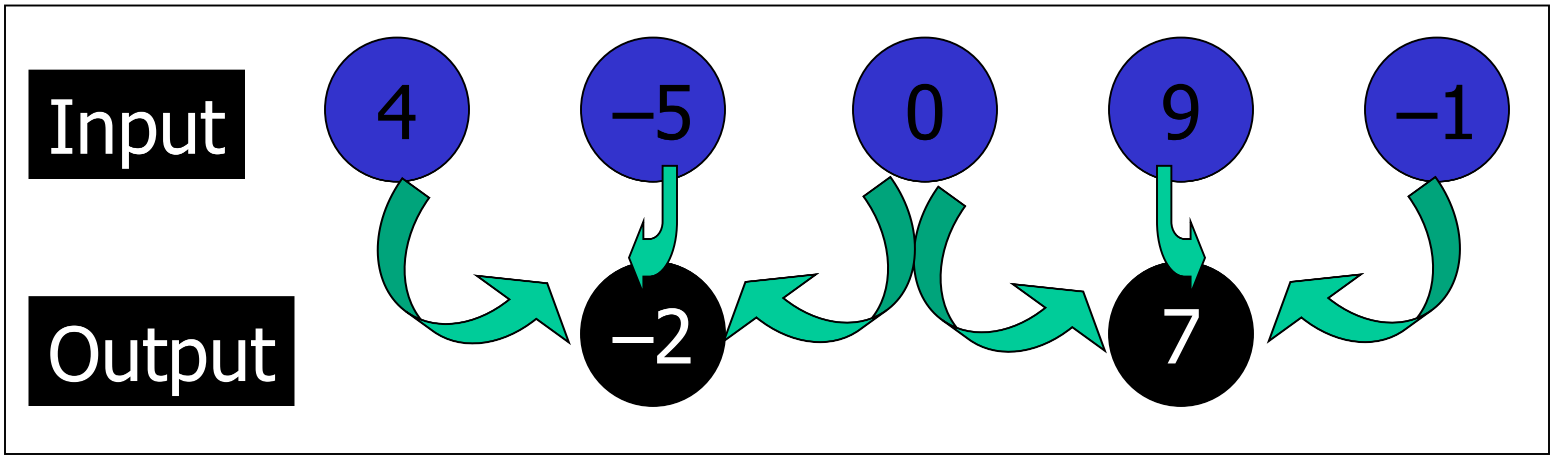
- What is the **domain**?

{0, 1, 2, 3, 4, 5}

What is the **range**?

{-5, -4, -3, -2, -1, 0}

Example 2



- What is the **domain**?

{4, -5, 0, 9, -1}

- What is the **range**?

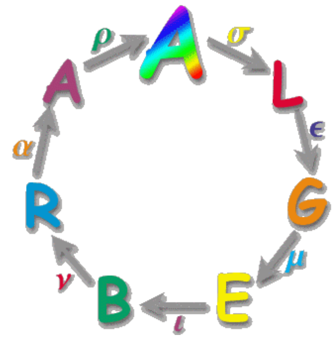
{-2, 7}

Is a relation a function?

What is a **function**?

According to the textbook, "**a function is**...**a relation in which every input is paired with exactly one output**"

Is a relation a function?



- Focus on the **x-coordinates**, when given a relation

If the set of ordered pairs have **different x-coordinates**,
it **IS A** function

If the set of ordered pairs have **same x-coordinates**,
it is **NOT** a function

- **Y-coordinates** have no bearing in determining functions

Example 3

$$\{(\mathbf{0},-\mathbf{5}),(\mathbf{1},-\mathbf{4}),(\mathbf{2},-\mathbf{3}),(\mathbf{3},-\mathbf{2}),(\mathbf{4},-\mathbf{1}),(\mathbf{5},\mathbf{0})\}$$

- ***Is this a function?***
  - ***Hint:*** *Look only at the* ***x-coordinates***

YES

Example 4

$$\{(-1,-7),(1,0),(2,-3),(0,-8),(0,5),(-2,-1)\}$$

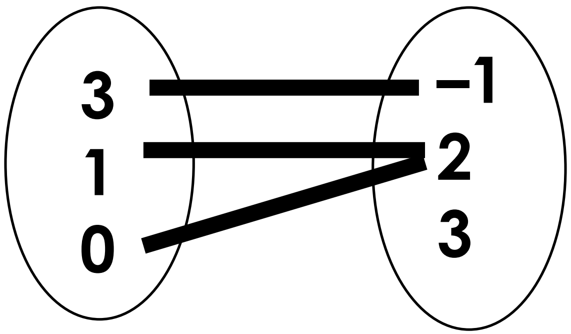
- ***Is this a function?***
  - ***Hint:*** *Look only at the **x-coordinates***

NO

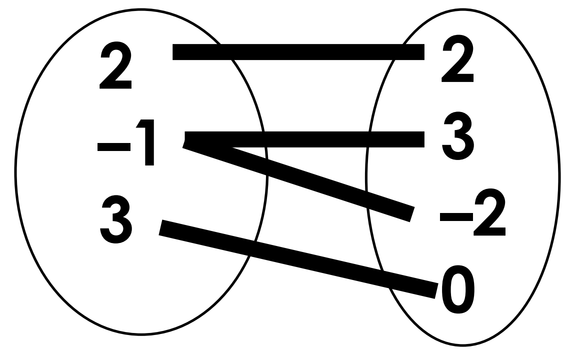
Example 5

Which mapping represents a function?

Choice One



Choice Two

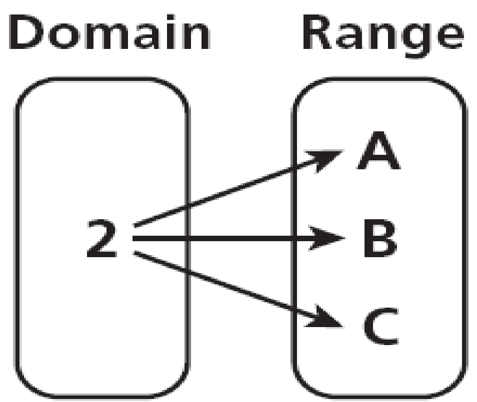


Choice 1

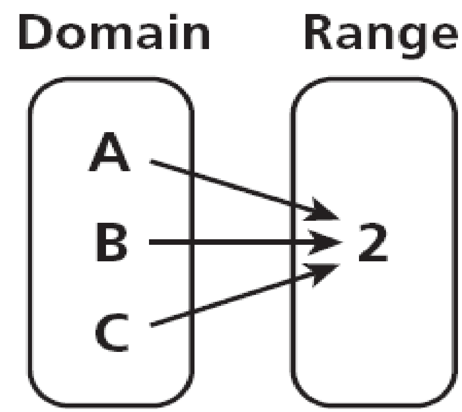
Example 6

Which mapping represents a function?

A.



B.



B

Example 7

Which situation represents a function?

a. **The items in a store to their prices on a certain date**

b. **Types of fruits to their colors**

There is only one price for each different item on a certain date. The relation from items to price makes it a function.

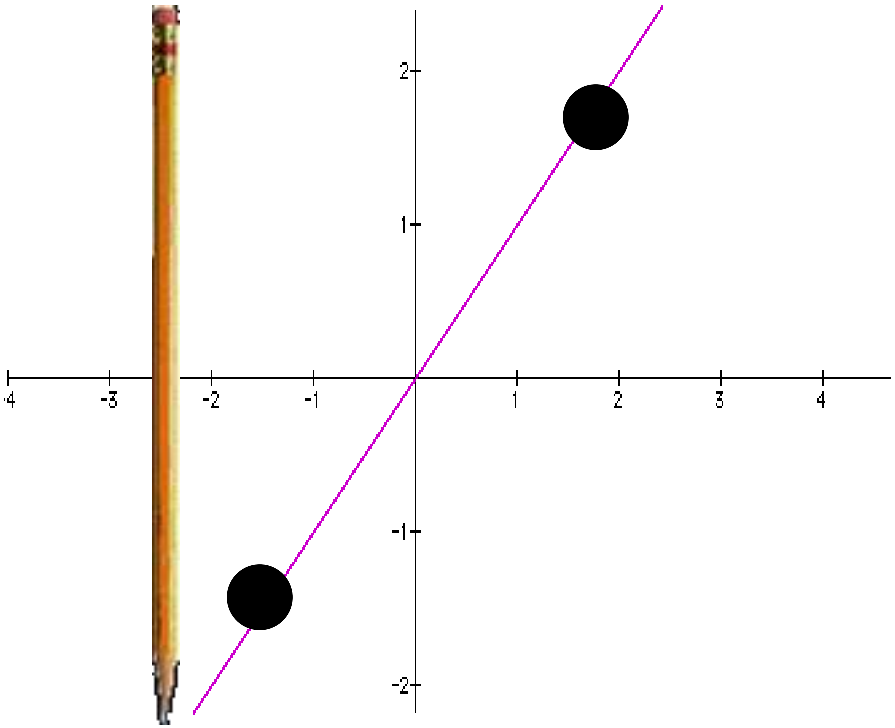
A fruit, such as an apple, from the domain would be associated with more than one color, such as red and green. The relation from types of fruits to their colors is not a function.

Vertical Line Test

- **Vertical Line Test:** *a relation is a function if a vertical line drawn through its graph, passes through only one point.*

*AKA: **"The Pencil Test"***
*Take a pencil and move it from **left to right (–x to x)**; if it crosses more than one point, it is not a function*

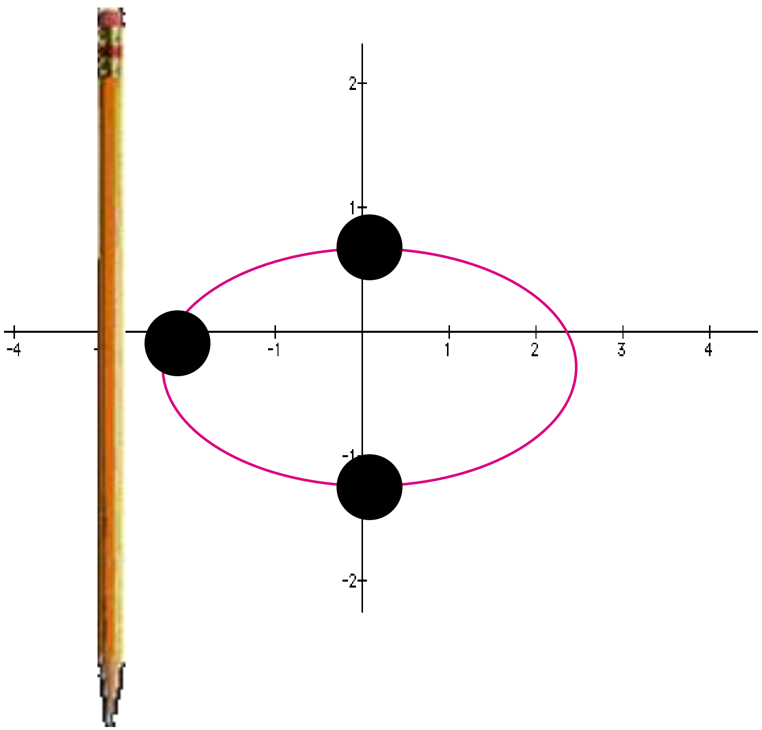
Vertical Line Test



Would this graph be a function?

YES

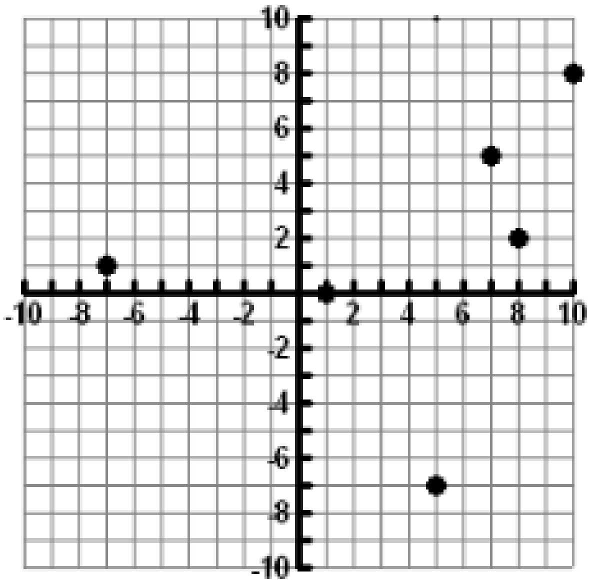
Vertical Line Test



Would this graph be a function?

NO

Is the following function discrete or continuous? What is the Domain? What is the Range?

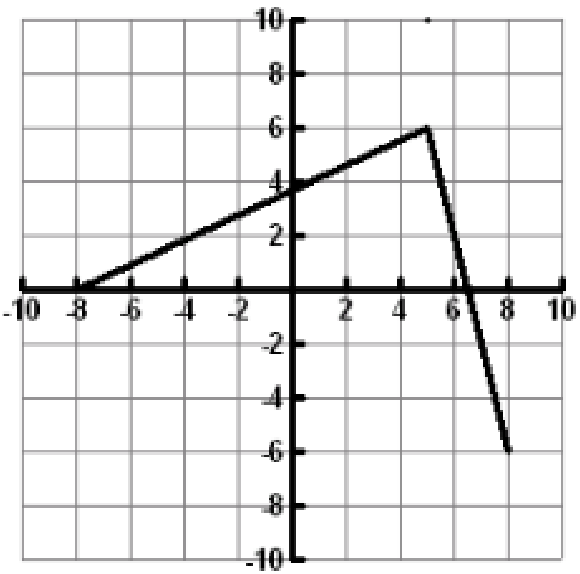


Type: Discrete

Domain: $\{-7, 1, 5, 7, 8, 10\}$

Range: $\{1, 0, -7, 5, 2, 8\}$

Is the following function discrete or continuous? What is the Domain? What is the Range?

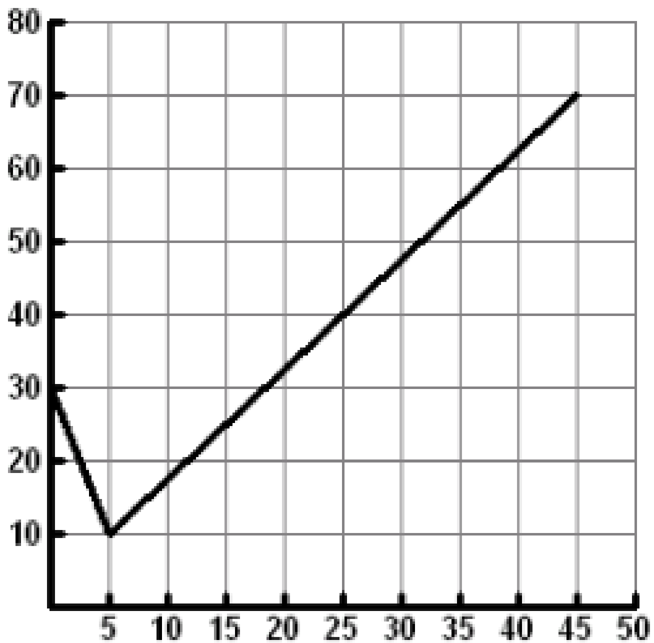


Type: continuous

Domain: $[-8,8]$

Range: $[-6,6]$

Is the following function discrete or continuous? What is the Domain? What is the Range?

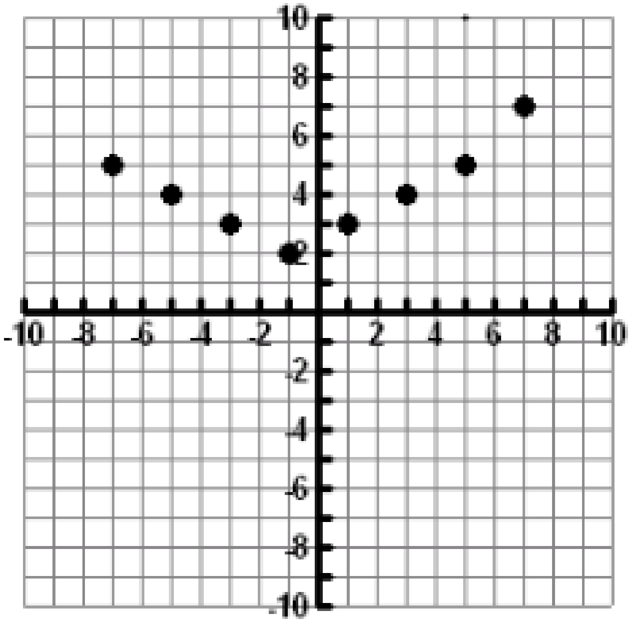


Type: continuous

Domain: [0,45]

Range: [10,70]

Is the following function discrete or continuous? What is the Domain? What is the Range?



Type: discrete

Domain: $\{-7, -5, -3, -1, 1, 3, 5, 7\}$

Range: $\{2, 3, 4, 5, 7\}$

Domain and Range in Real Life

The number of shoes in x **pairs** of shoes can be expressed by the equation y = 2x.

What subset of the real numbers makes sense for the domain?

Whole numbers

What would make sense for the range of the function?

Zero and the even numbers

Domain and Range in Real Life

The number of shoes in x **pairs** of shoes can be expressed by the equation y = 2x.

What is the independent variable?

The **# of pairs** of shoes.

What is the dependent variable?

The **total #** of shoes.

Domain and Range in Real Life

Mr. Landry is driving to his hometown. It takes four hours to get there. The distance he travels at any time, *t*, is represented by the function d = 55t (his average speed is 55mph.

Write an inequality that represents the domain in real life.

$$0 \le x \le 4$$

Write an inequality that represents the range in real life.

$$0 \le y \le 220$$

Domain and Range in Real Life

Mr. Landry is driving to his hometown. It takes four hours to get there. The distance he travels at any time, *t*, is represented by the function d = 55t (his average speed is 55mph.

What is the independent variable?

The time that he drives.

What is the dependent variable?

The total distance traveled.

Domain and Range in Real Life

Johnny bought at most 10 tickets to a concert for him and his friends. The cost of each ticket was $12.50. Complete the table below to list the possible domain and range.

1	2	3	4	5	6	7	8	9	10
12.50	25.00	37.50	50	62.50	75	87.50	100	112.50	125

What is the independent variable?

The number of tickets bought.

What is the dependent variable?

The total cost of the tickets.

Domain and Range in Real Life

Pete’s Pizza Parlor charges $5 for a large pizza with no toppings. They charge an additional $1.50 for each of their 5 specialty toppings (tax is included in the price).

Jorge went to pick up his order. They said his total bill was $9.50. Could this be correct? Why or why not?

Yes   One pizza with 3 toppings cost $9.50

Susan went to pick up her order. They said she owed $10.25. Could this be correct? Why or why not?

No   One pizza with 4 toppings cost $11

Domain and Range in Real Life

Pete's Pizza Parlor charges $5 for a large pizza with no toppings. They charge an additional $1.50 for each of their 5 specialty toppings (tax is included in the price).

What is the independent variable?

The number of toppings

What is the dependent variable?

The cost of the pizza

Function Notation

$f(x)$ means function of x and is read "f of x."

$f(x) = 2x + 1$ is written in function notation.

The notation $f(1)$ means to replace x with 1 resulting in the function value.

$f(1) = 2x + 1$
$f(1) = 2(1) + 1$
$f(1) = 3$

Function Notation

Given $g(x) = x^2 - 3$, find $g(\text{-}2)$.

$$g(-2) = x^2 - 3$$
$$g(-2) = (-2)^2 - 3$$
$$g(-2) = 1$$

Function Notation

Given $f(x) = 2x^2 - 3x$, the following.

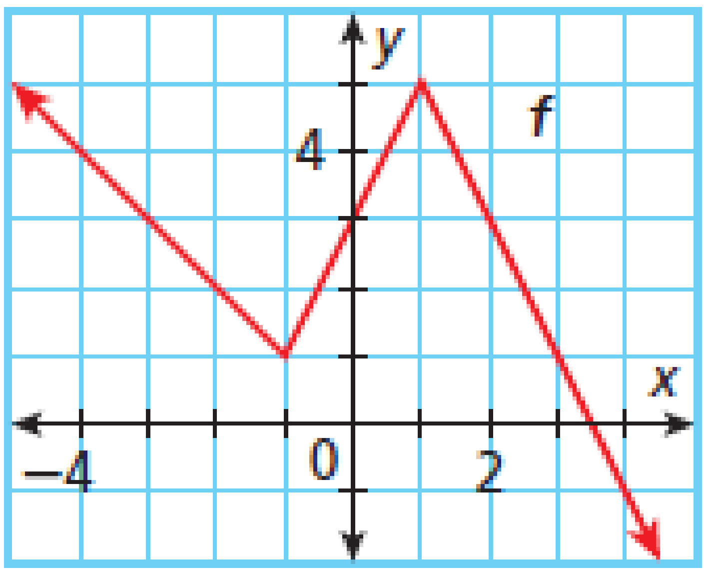
a. $f(3)$ **b.** $3f(x)$ **c.** $f(3x)$

$f(3) = 2x^2 - 3x$
$f(3) = 2(3)^2 - 3(3)$
$f(3) = 2(9) - 9$
$f(3) = 9$

$3f(x) = 3(2x^2 - 3x)$
$3f(x) = 6x^2 - 9x$

$f(3x) = 2x^2 - 3x$
$f(3x) = 2(3x)^2 - 3(3x)$
$f(3x) = 2(9x^2) - 3(3x)$
$f(3x) = 18x^2 - 9x$

For each function, evaluate $f(0)$, $f(1.5)$, $f(-4)$,

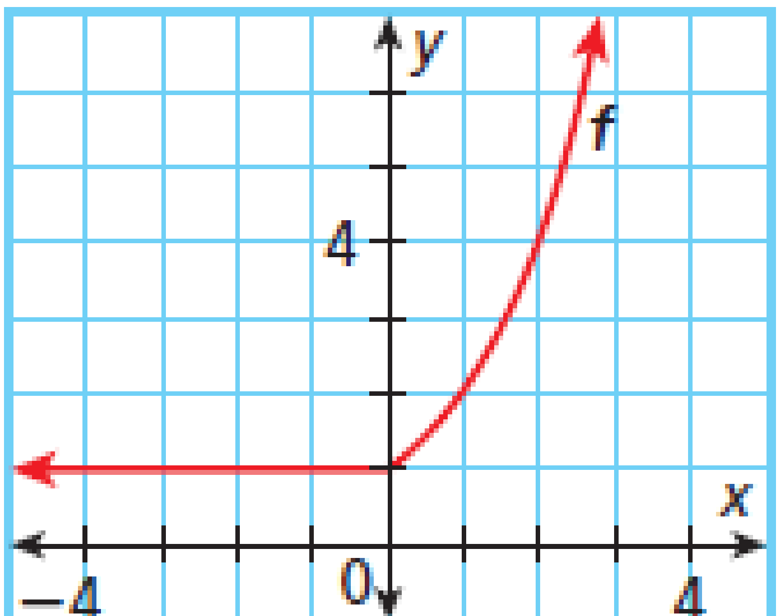


$f(0) =$ 3

$f(1.5) =$ 4

$f(-4) =$ 4

For each function, evaluate $f(0)$, $f(1.5)$, $f(-4)$,

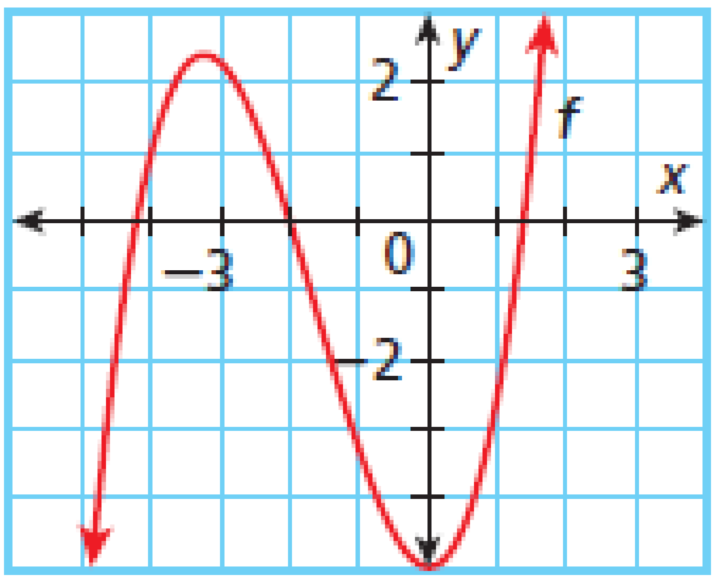


$f(0) =$ 1

$f(1.5) =$ 3

$f(-4) =$ 1

For each function, evaluate $f(0)$, $f(1.5)$, $f(-4)$,



$f(0) =$ -5

$f(1.5) =$ 1

$f(-4) =$ 1

Practice!

Pg. 244, 245 #1-41 odd

#43-48 (Can be turned in for 6 extra credit points!!)

Graphing a Function Rule

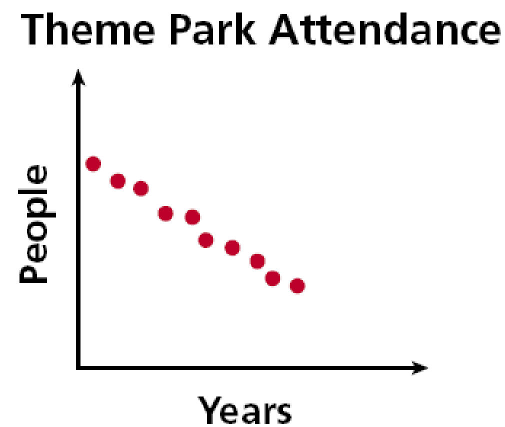
Section 5-3

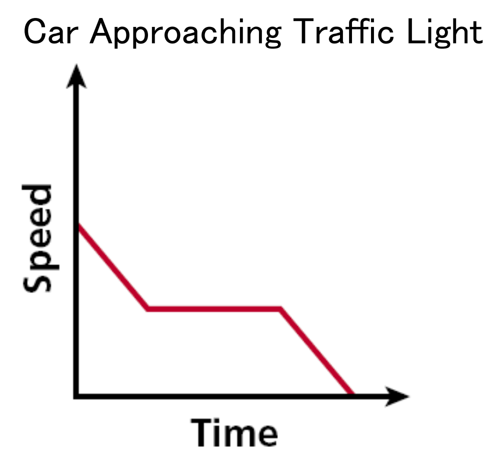
Vocabulary

- Continuous graph
- Discrete graph

Definition

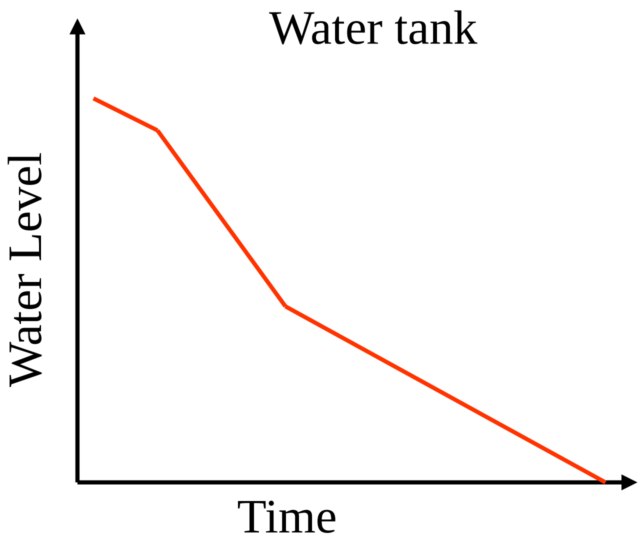
- Some graphs are connected lines or curves called *continuous graphs*. Some graphs are only distinct points. They are called *discrete graphs*.
- Example discrete graphs:
  – The graph on theme park attendance is an example of a discrete graph. It consists of distinct points because each year is distinct and people are counted in whole numbers only. The values between whole numbers are not included, since they have no meaning for the situation.
- Example continuous graphs:
  – The graph of a car approaching a traffic light is an example of a continuous graph. It consists of continuous line and all the points on the line, because any point on the line has meaning.





Example:

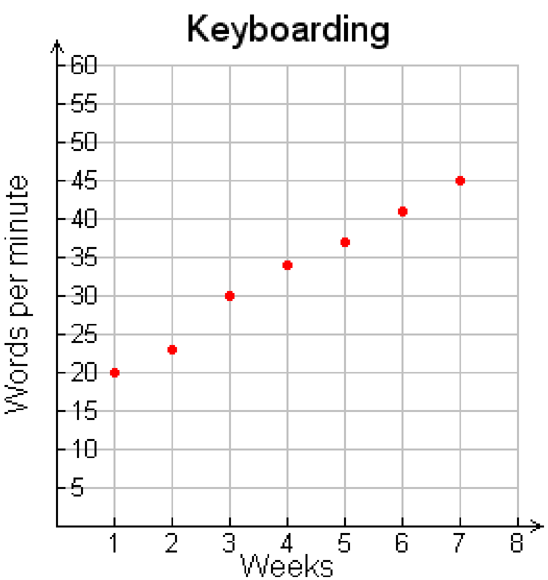
Henry begins to drain a water tank by opening a valve. Tell whether the graph is continuous or discrete.



The graph is continuous.

Example:

Jamie is taking an 8-week keyboarding class. At the end of each week, she takes a test to find the number of words she can type per minute. Tell whether the graph is continuous or discrete.



The graph is discrete.

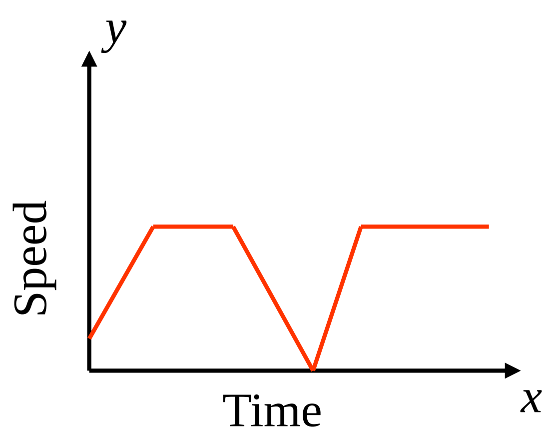
Your Turn:

A small bookstore sold between 5 and 8 books each day for 7 days. Tell whether the graph is continuous or discrete.

The graph is discrete.

Your Turn:

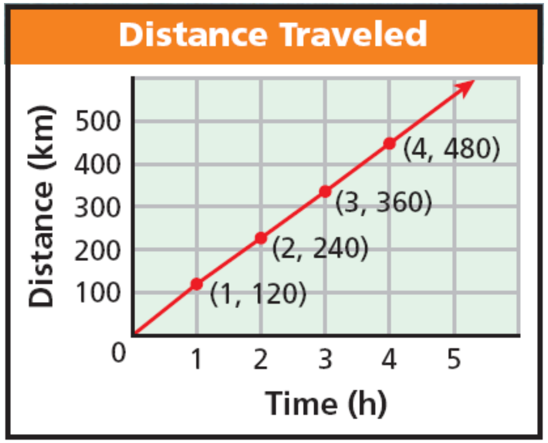
A truck driver enters a street, drives at a constant speed, stops at a light, and then continues. Tell whether the graph is continuous or discrete.



The graph is continuous.

Graphing Linear Functions

Many stretches on the German autobahn have a speed limit of 120 km/h. If a car travels continuously at this speed, $y = 120x$ gives the number of kilometers y that the car would travel in x hours.



Notice that the graph is a straight line. An equation whose graph forms a straight line is a *linear equation*. Also notice that this is a function. A function represented by a linear equation is a *linear function*.

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line. However, graphing three points is a good way to check that your line is correct.

Graphing

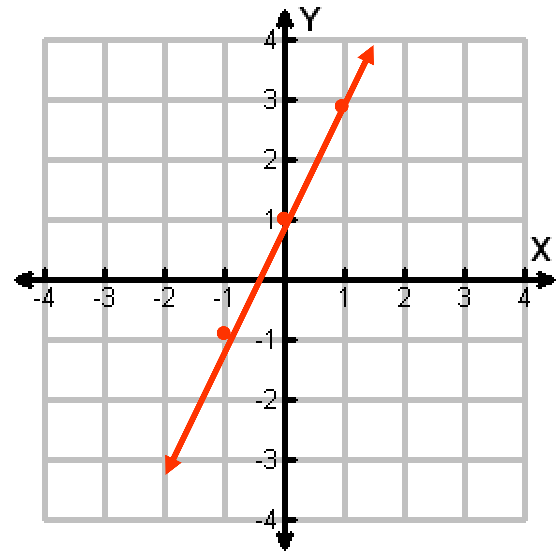
Procedure: Graphing Functions	
Step 1	Use the function to generate ordered pairs by choosing several values for x.
Step 2	Plot enough points to see a pattern for the graph.
Step 3	Connect the points with a line or smooth curve.

Example: Graphing Linear Functions

Graph $y = 2x + 1$.

Step 1 Choose **three values of x** and generate ordered pairs.

x	$y = 2x + 1$	(x, y)
1	$y = 2(1) + 1 = 3$	$(1, 3)$
0	$y = 2(0) + 1 = 1$	$(0, 1)$
–1	$y = 2(–1) + 1 = –1$	$(–1, –1)$



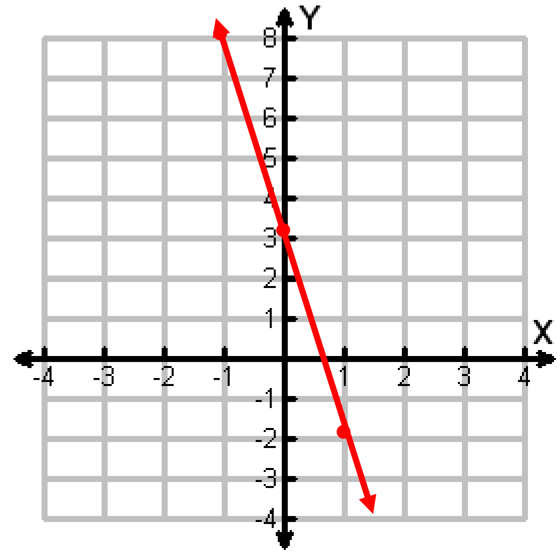
Step 2 Plot the points and connect them with a straight line.

Example: Graphing Linear Functions

Graph 15*x* + 3*y* = 9.

Step 1 Choose **three values of *x*** and generate ordered pairs

x	$y = -5x + 3$	(x, y)
1	$y = -5(1) + 3 = -2$	$(1, -2)$
0	$y = -5(0) + 3 = 3$	$(0, 3)$
–1	$y = -5(-1) + 3 = 8$	$(-1, 8)$



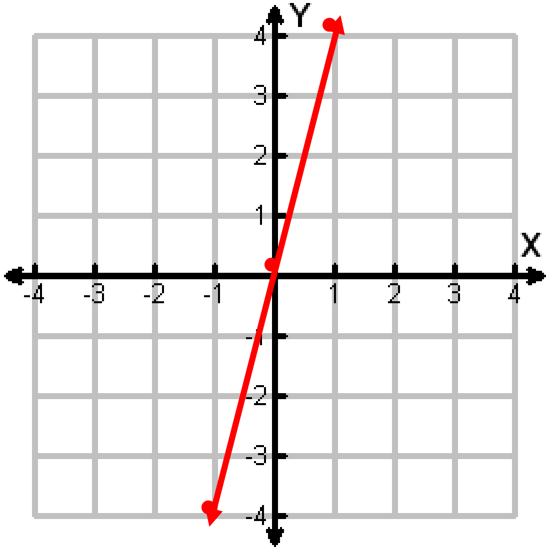
Step 2 Plot the points and connect them with a straight line.

Your Turn:

Graph *y* = 4*x*.

Step 1 Choose **three values of *x*** and generate ordered pairs

x	$y = 4x$	(x, y)
1	$y = 4(1) = 4$	(1, 4)
0	$y = 4(0) = 0$	(0, 0)
–1	$y = 4(–1) = –4$	(–1, –4)



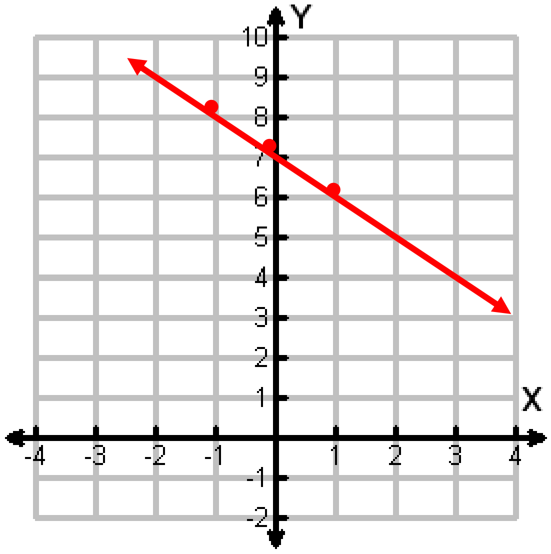
Step 2 Plot the points and connect them with a straight line.

Your Turn:

Graph $y + x = 7$.

Step 2 Choose **three values of x** and generate ordered pairs

x	$y = -x + 7$	(x, y)
1	$y = -(1) + 7 = 6$	$(1, 6)$
0	$y = -(0) + 7 = 7$	$(0, 7)$
-1	$y = -(-1) + 7 = 8$	$(-1, 8)$



Step 3 Plot the points and connect them with a straight line.

Graphing Real-World Functions

- In many real-world situations, the *x* and *y* values must be restricted. For example, quantities such as time, distance, and number of people can be represented using only nonnegative values.
- Sometimes *x* and *y* values are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

Example: Application

The relationship between human years and dog years is given by the function $y = 7x$, where x is the number of human years. Graph this function.

Choose several values of x and make a table of ordered pairs.

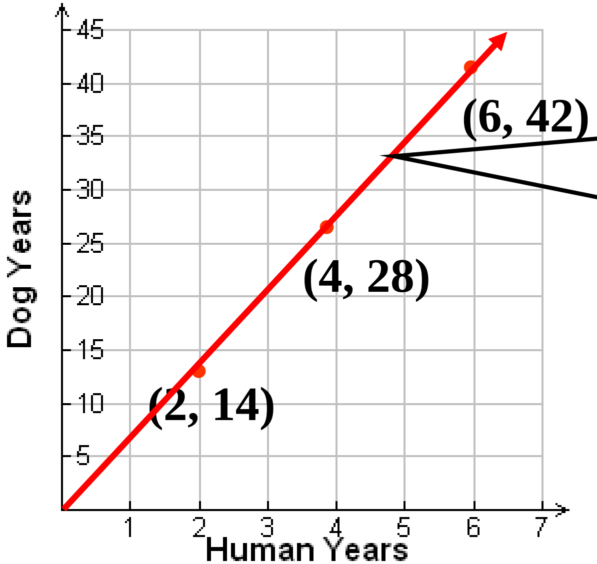
x	$y = 7x$	(x, y)
2	$y = 7(2) = 14$	(2, 14)
4	$y = 7(4) = 28$	(4, 28)
6	$y = 7(6) = 42$	(6, 42)

The ages are continuous starting with 0.

Example: Continued

Graph the ordered pairs.

Human Years vs. Dog Years



Any point on the line is a solution in this situation. The arrow shows that the trend continues.

Your Turn:

At a salon, Sue can rent a station for \$10.00 per day plus \$3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where x is the number of manicures. Graph this function.

Your Turn: Solution

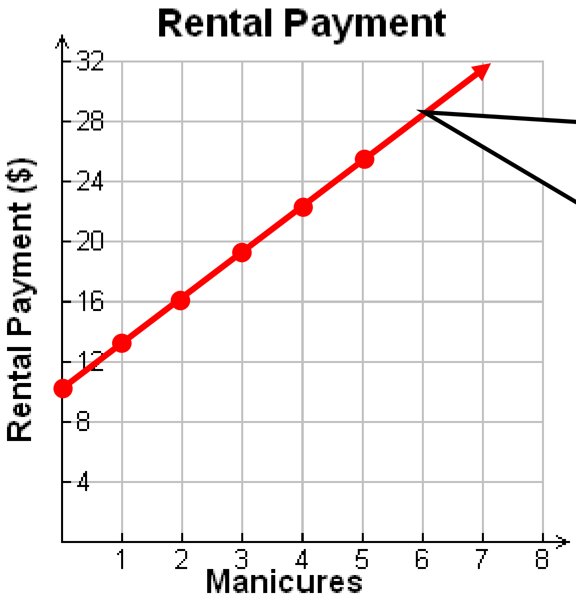
Choose several values of x and make a table of ordered pairs.

x	$f(x) = 3x + 10$
0	$f(0) = 3(0) + 10 = 10$
1	$f(1) = 3(1) + 10 = 13$
2	$f(2) = 3(2) + 10 = 16$
3	$f(3) = 3(3) + 10 = 19$
4	$f(4) = 3(4) + 10 = 22$
5	$f(5) = 3(5) + 10 = 25$

The number of manicures must be a whole number.

Your Turn: Solution

Graph the ordered pairs.



The individual points are solutions in this situation. The line shows that the trend continues.

Example: Graphing Nonlinear Functions

Graph the function $g(x) = |x| + 2$.

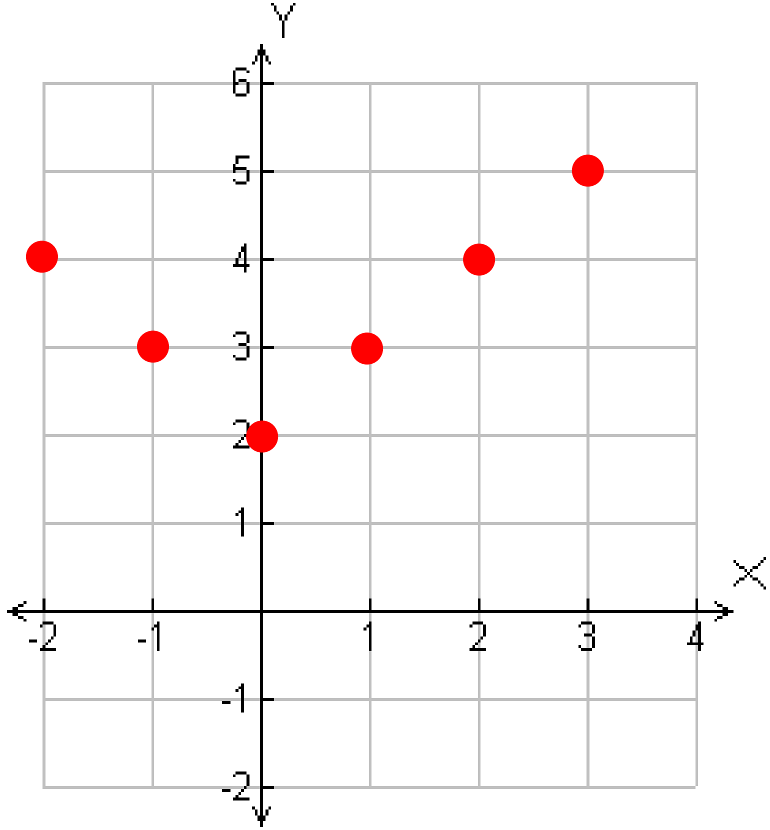
Step 1 Choose several values of x and generate ordered pairs.

x	$g(x) = \|x\| + 2$	$(x, g(x))$
–2	$g(x) = \|-2\| + 2 = 4$	(–2, 4)
–1	$g(x) = \|-1\| + 2 = 3$	(–1, 3)
0	$g(x) = \|0\| + 2 = 2$	(0, 2)
1	$g(x) = \|1\| + 2 = 3$	(1, 3)
2	$g(x) = \|2\| + 2 = 4$	(2, 4)
3	$g(x) = \|3\| + 2 = 5$	(3, 5)

Example: Continued

Graph the function $g(x) = |x| + 2$.

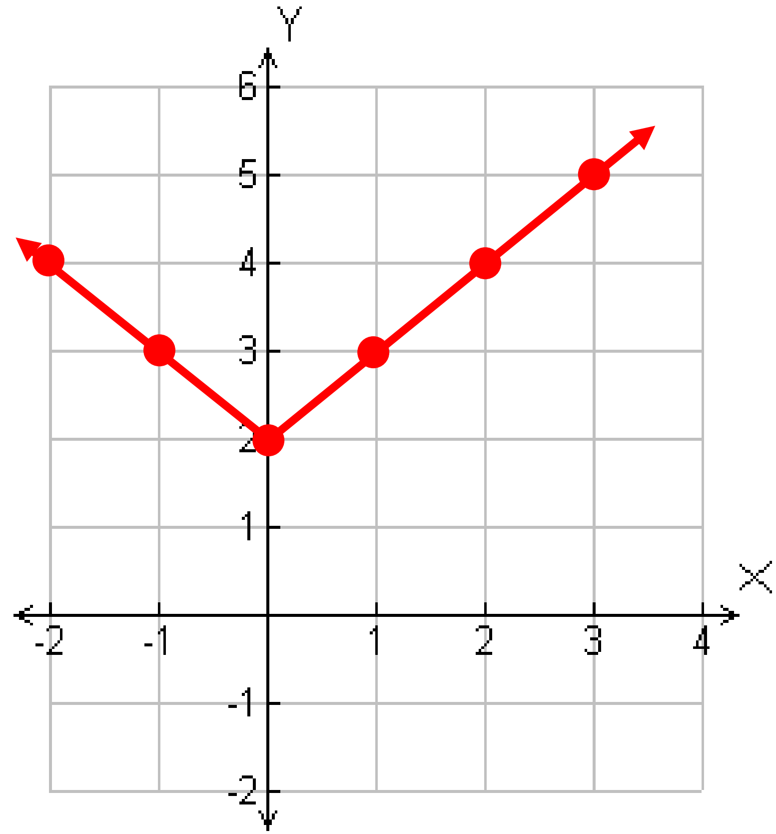
Step 2 Plot enough points to see a pattern.



Example: Continued

Graph the function *g*(*x*) = |*x*| + 2.

Step 3 The ordered pairs appear to form a v-shape. Draw lines through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on the “ends” of the “v”.



Your Turn:

Graph the function $y = |x - 1|$.

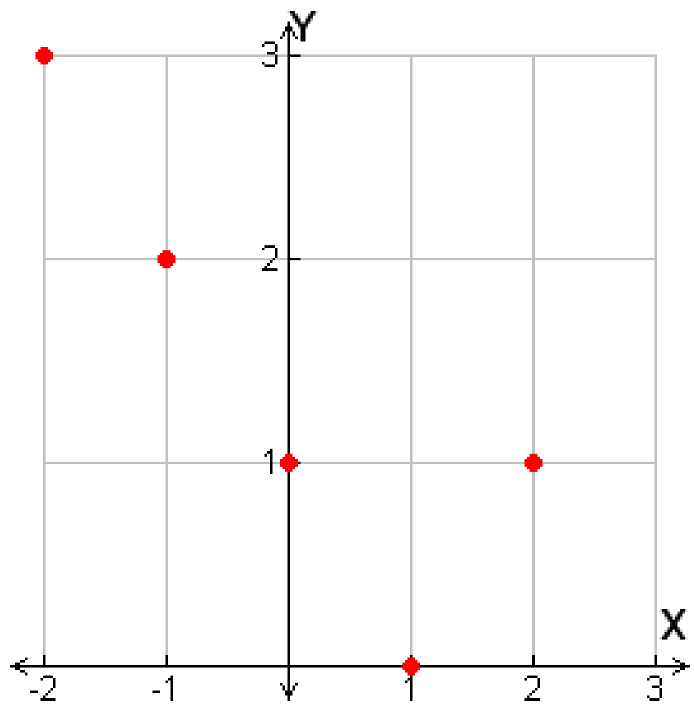
Step 1 Choose several values of x and generate ordered pairs.

x	$y = \|x - 1\|$	(x, y)
-2	$y = \|-2 - 1\| = 3$	$(-2, 3)$
-1	$y = \|-1 - 1\| = 2$	$(-1, 2)$
0	$y = \|0 - 1\| = 1$	$(0, 1)$
1	$y = \|1 - 1\| = 0$	$(1, 0)$
2	$y = \|2 - 1\| = 1$	$(2, 1)$

Continued

Graph the function *y* = |*x* – 1|.

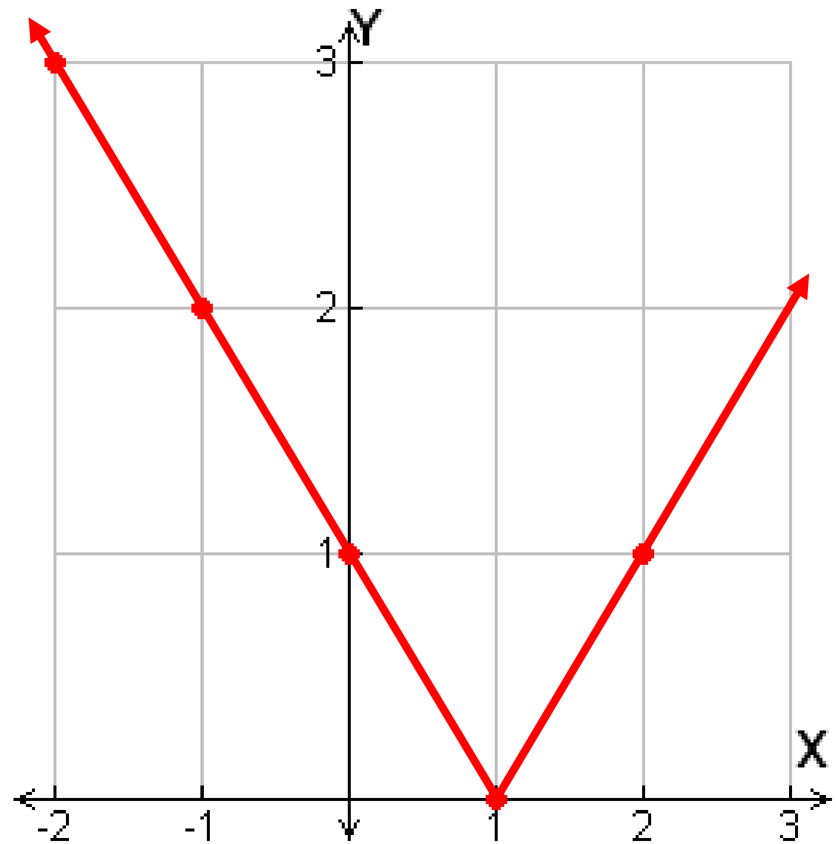
Step 2 Plot enough points to see a pattern.



Continued

Graph the function *y* = |*x* – 1|.

Step 3 The ordered pairs appear to form a V-shape. Draw a line through the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both “ends” of the “V”..



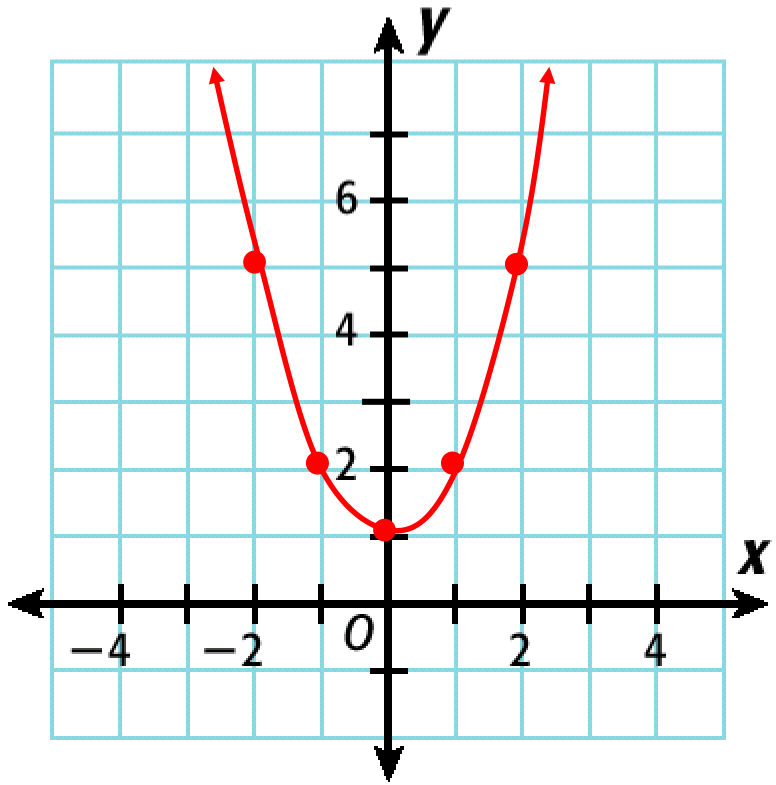
Your Turn:

Graph the function $y = x^2 + 1$.

Solution:

$y = x^2 + 1$

x	$x^2 + 1$	y
−2	$(-2)^2 + 1$	5
−1	$(-1)^2 + 1$	2
0	$(0)^2 + 1$	1
1	$(1)^2 + 1$	2
2	$(2)^2 + 1$	5



Plot the points and connect them with a smooth curve.

Assignment

- Pg. 249-250 #1-11 odd, #13, #15-23 all, #38

Writing a Function Rule

Section 5-4

Writing a Function Rule

- An algebraic expression that defines a function is a *function rule*.
- The dependent variable is a function of the independent variable.
- *y* is a function of *x*.

Example: Writing a Function Rule

- Write a function rule that represents each sentence.

A worker's earnings *e* are a function of the number of hours *n* worked at a rate of $8.75 per hour.

earnings	are	number of hours worked	at a rate **OF**	$8.75/hr.
e	$=$	n	$\bullet$	8.75

A function rule that represents this situation is: ***e* = 8.75*n***

Your Turn:

- A helicopter hovers 40ft above the ground. Then, the helicopter climbs at a rate of 21ft/s. Write a rule that represents the helicopter's height *h* above the ground as a function of time *t*.

height	is	40ft	increased by	21ft	per	second
h	$=$	40	$+$	21	$\bullet$	t

A function rule that represents this situation is: $\boldsymbol{h = 21t + 40}$

Example: Writing Functions

Identify the independent and dependent variables. Write a function rule for the situation.

A math tutor charges $35 per hour.

The amount a math tutor charges depends on number of hours.

Dependent: charges
Independent: hours

Let C represent the charges.
Let h represent the number of hours of tutoring.

The function for the amount a math tutor charges is $C = 35h$.

Example: Writing Functions

Identify the independent and dependent variables. Write a function rule for the situation.

A fitness center charges a $100 initiation fee plus $40 per month.

The total cost depends on the number of months, plus $100.

Dependent: total cost
Independent: number of months

Let C represent the total cost.
Let m represent the number of months

The function for the amount the fitness center charges is $C = 40m + 100$.

Your Turn:

Identify the independent and dependent variables. Write a function rule for the situation.

Steven buys lettuce that costs \$1.69/lb.

The total cost depends on how many pounds of lettuce that Steven buys.

Dependent: total cost
Independent: pounds

Let C represent the total cost.
Let x represent the number of pounds Steven bought.

The function for cost of the lettuce is $C = 1.69x$.

Your Turn:

Identify the independent and dependent variables. Write a function rule for the situation.

An amusement park charges a $6.00 parking fee plus $29.99 per person.

The total cost depends on the number of persons in the car, plus $6.

Dependent: total cost
Independent: number of persons in the car

Let C represent the total cost.
Let x represent the number of persons in the car.

The function for the total park cost is $C = 29.99x + 6$.

Assignment

- Pg. 256-257 #1-16 all, #18, 21, 27

Formalizing Relations and Functions

Unit 5

Vocabulary

- Relation
- Domain
- Range
- Vertical line test
- Function notation

Definition

- *Relation* – A relationships that can be represented by a set of ordered pairs.
- Example:
  - In the scoring systems of some track meets, for first place you get 5 points, for second place you get 3 points, for third place you get 2 points, and for fourth place you get 1 point. This scoring system is a relation, so it can be shown by ordered pairs. {(1, 5), (2, 3), (3, 2) (4, 1)}.
- You can also show relations in other ways, such as tables, graphs, or *mapping diagrams*.

Example: Relations

Express the relation {(2, 3), (4, 7), (6, 8)} as a table, as a graph, and as a mapping diagram.

Table

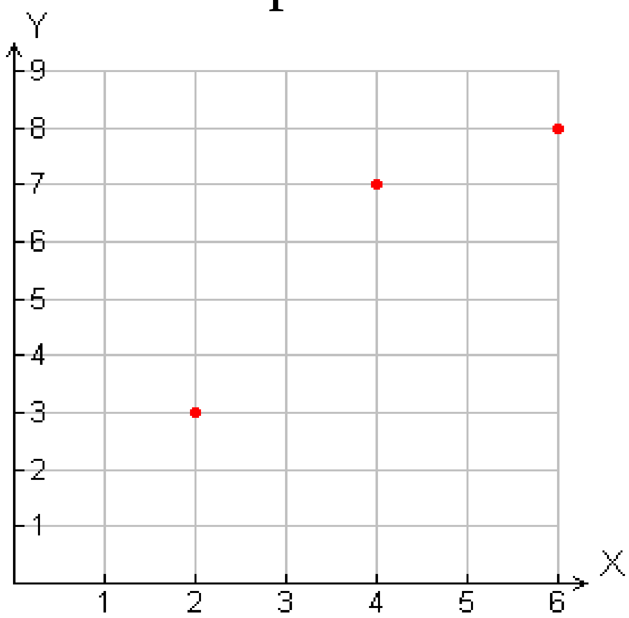
x	y
2	3
4	7
6	8

Write all x-values under "x" and all y-values under "y".

Example: Continued

Express the relation {(2, 3), (4, 7), (6, 8)} as a table, as a graph, and as a mapping diagram.

Graph

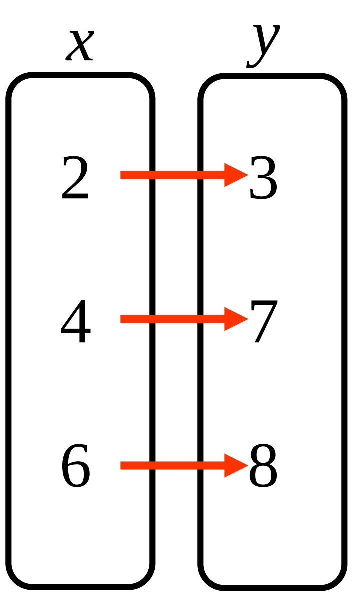


Use the x- and y-values to plot the ordered pairs.

Example: Continued

Express the relation {(2, 3), (4, 7), (6, 8)} as a table, as a graph, and as a mapping diagram.

Mapping Diagram

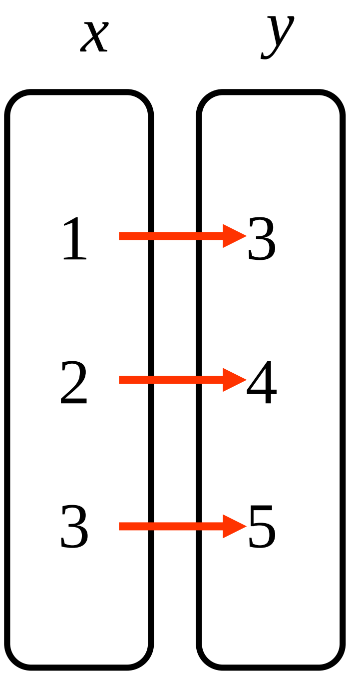


Write all x-values under "x" and all y-values under "y". Draw an arrow from each x-value to its corresponding y-value.

Your Turn:

Express the relation {(1, 3), (2, 4), (3, 5)} as a table, as a graph, and as a mapping diagram.

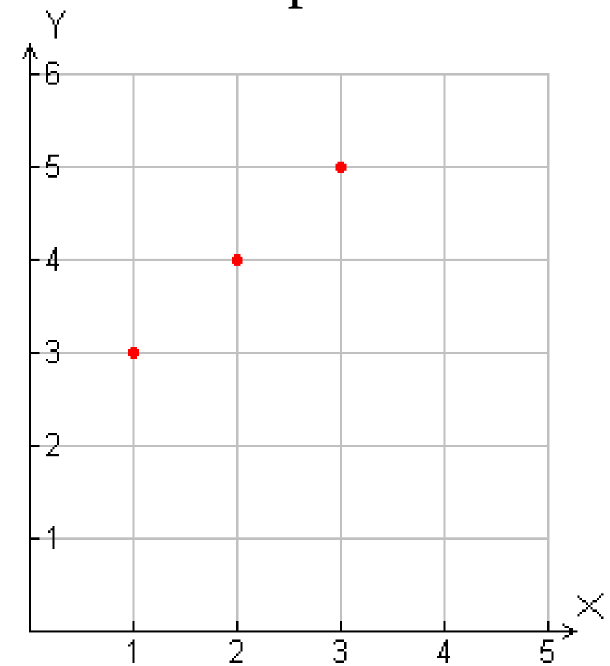
Mapping Diagram



Table

x	y
1	3
2	4
3	5

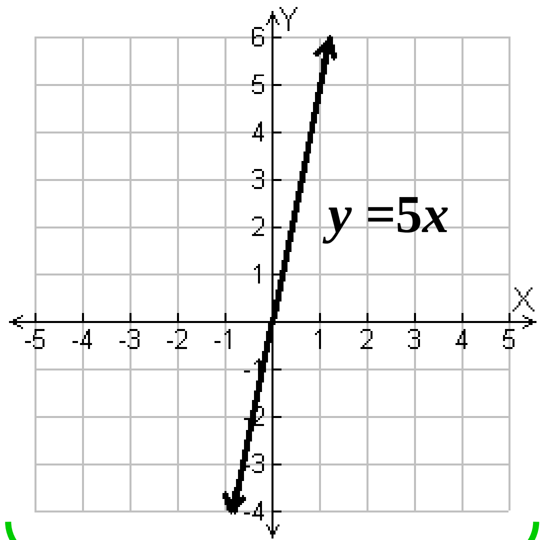
Graph



Definition

- *Domain* – The set of first coordinates (or *x*-values) of the ordered pairs of a relation.
- Example:
  - For the track meet scoring system relation, {(1, 5), (2, 3), (3, 2) (4, 1)}. The domain of the track meet scoring system is {1, 2, 3, 4}.
- *Range* - The set of second coordinates (or *y*-values) of the ordered pairs of a relation.
  - For the track meet scoring system relation, {(1, 5), (2, 3), (3, 2) (4, 1)}. The range is {5, 3, 2, 1}.

Example: Finding Domain and Range from a Graph



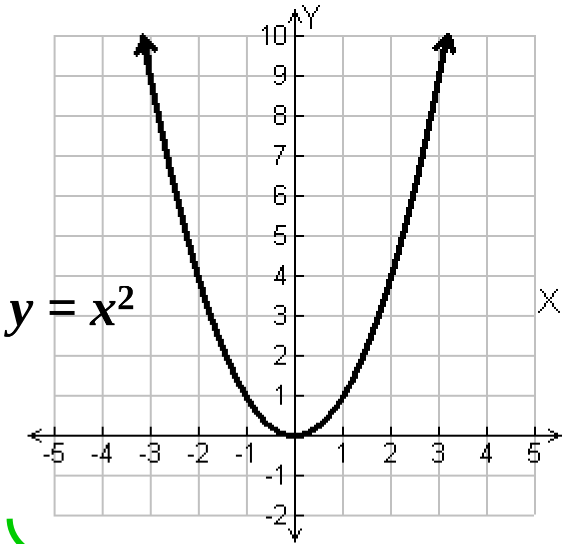
All *y*-values appear somewhere on the graph.

All *x*-values appear somewhere on the graph.

For $y = 5x$ the domain is all real numbers and the range is all real numbers.

Example: Finding Domain and Range from a Graph

$y = x^2$



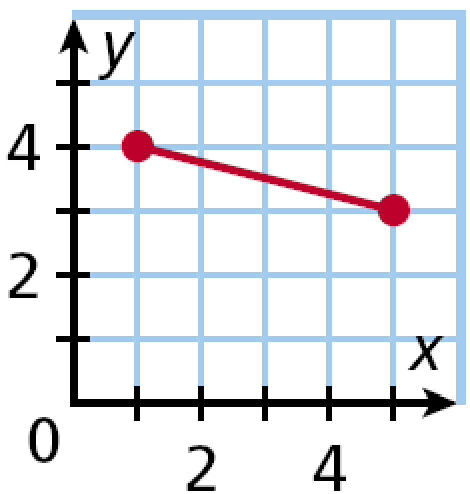
Only nonnegative *y*-values appear on the graph.

All *x*-values appear somewhere on the graph.

For $y = x^2$ the domain is all real numbers and the range is $y \geq 0$.

Your Turn:

Give the domain and range of the relation.



The domain value is all *x*-values from 1 through 5, inclusive.

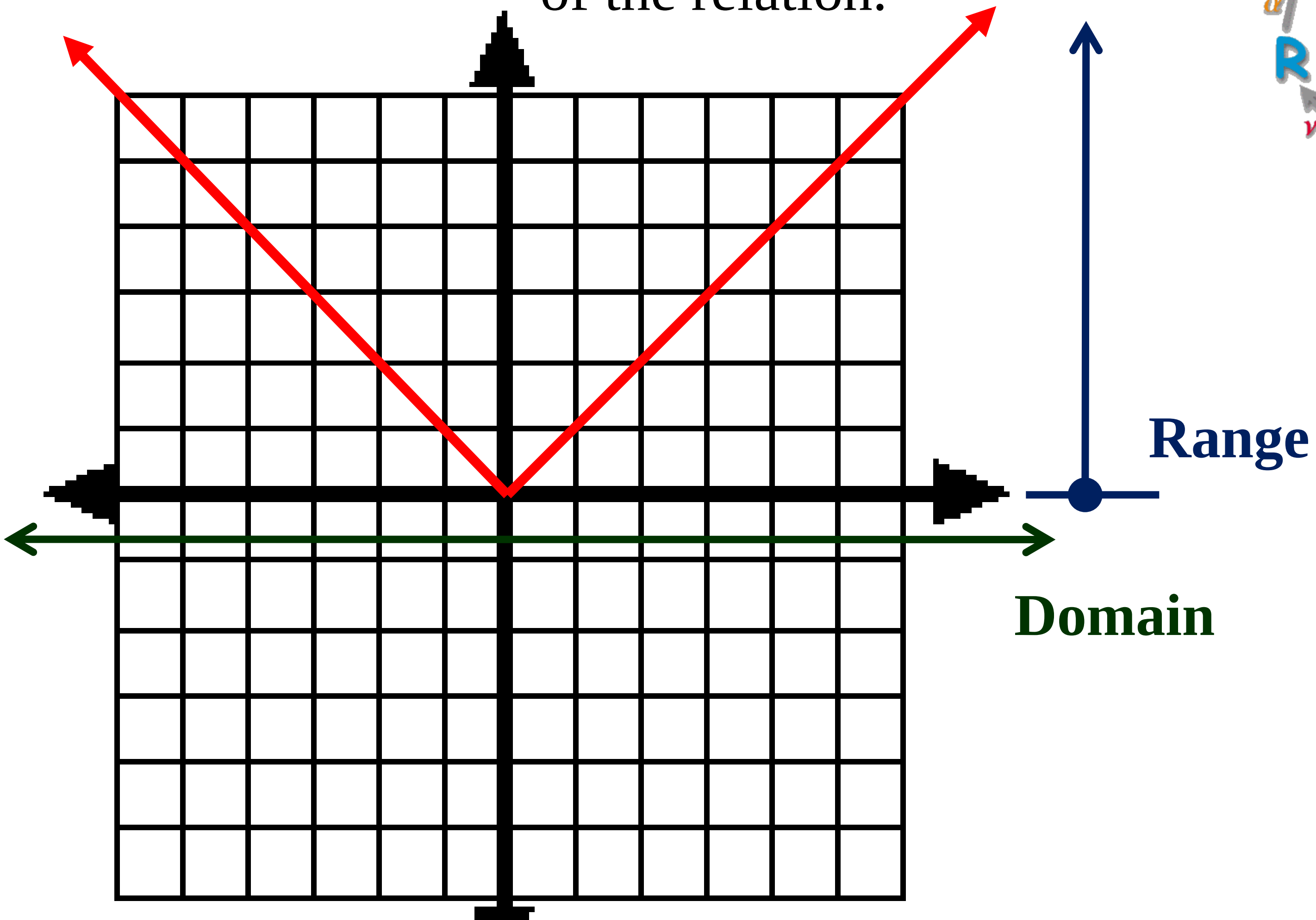
The range value is all *y*-values from 3 through 4, inclusive.

Domain: $1 \leq x \leq 5$

Range: $3 \leq y \leq 4$

Your Turn:

Give the domain and range of the relation.

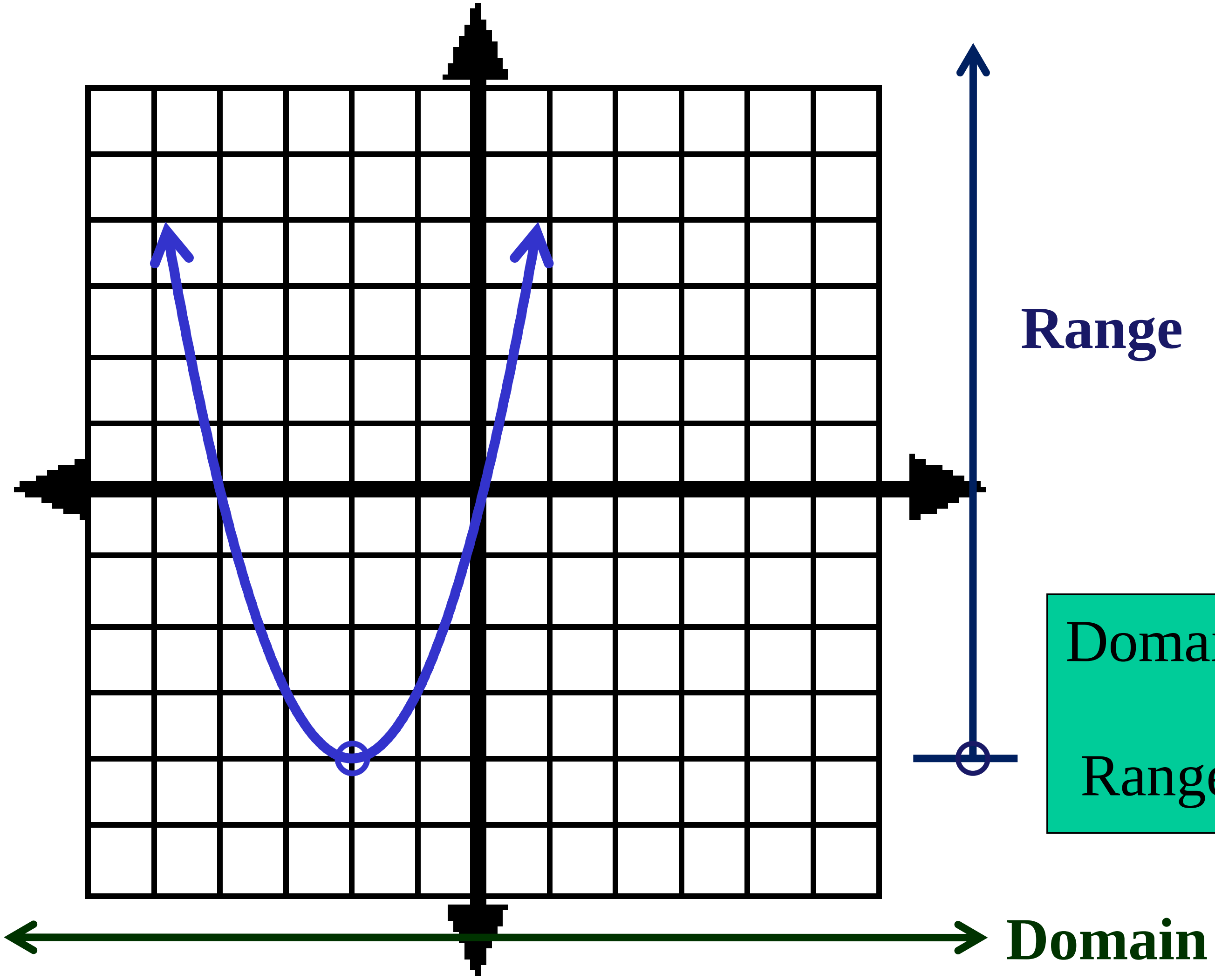


Domain: all real numbers

Range: $y \geq 0$

Your Turn:

Give the domain and range of the relation.

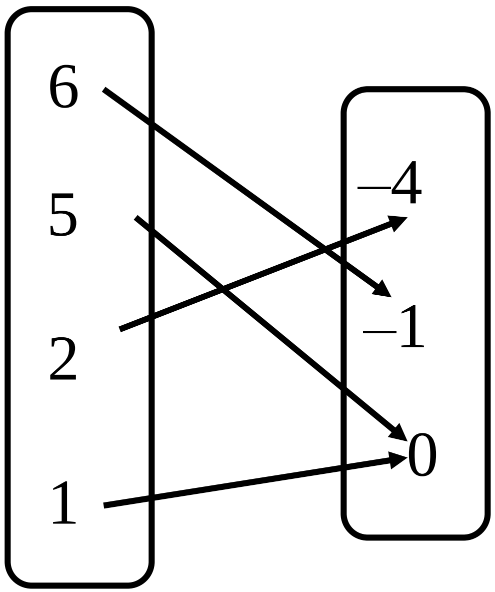


Domain: all real numbers

Range: y > -4

Your Turn:

Give the domain and range of the relation.



The domain values are all *x*-values 1, 2, 5 and 6.

The range values are *y*-values 0, –1 and –4.

Domain: {6, 5, 2, 1}

Range: {–4, –1, 0}

Your Turn:

Give the domain and range of the relation.

x	*y*
1	1
4	4
8	1

The domain values are all x-values 1, 4, and 8.

The range values are y-values 1 and 4.

Domain: {1, 4, 8}

Range: {1, 4}

Functions

- A *function* is a special type of relation that pairs each domain value with exactly one range value.
- All functions are relations, but all relations are not functions.

Example:

Give the domain and range of the relation. Tell whether the relation is a function. Explain.

{(3, –2), (5, –1), (4, 0), (3, 1)}

D: {3, 5, 4}

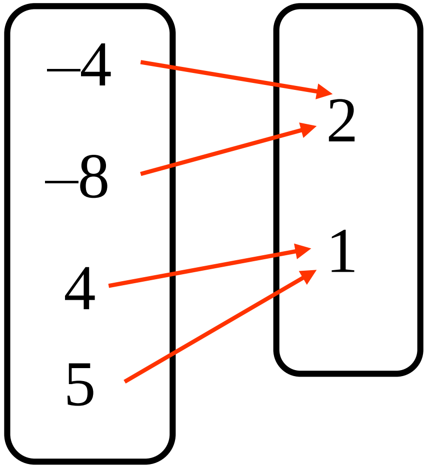
R: {–2, –1, 0, 1}

Even though 3 is in the domain twice, it is written only once when you are giving the domain.

The relation is not a function. Each domain value does not have exactly one range value. The domain value 3 is paired with the range values –2 and 1.

Example:

Give the domain and range of the relation. Tell whether the relation is a function. Explain.



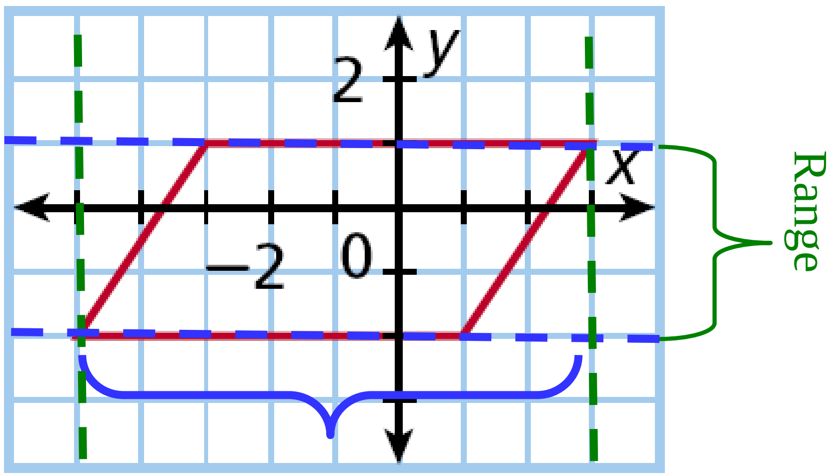
Use the arrows to determine which domain values correspond to each range value.

D: {–4, –8, 4, 5}   R: {2, 1}

This relation is a function. Each domain value is paired with exactly one range value.

Example:

Give the domain and range of the relation. Tell whether the relation is a function. Explain.



Domain

Draw in lines to see the domain and range values

D: $-5 \leq x \leq 3$ R: $-2 \leq y \leq 1$

The relation is not a function. Nearly all domain values have more than one range value.

Your Turn:

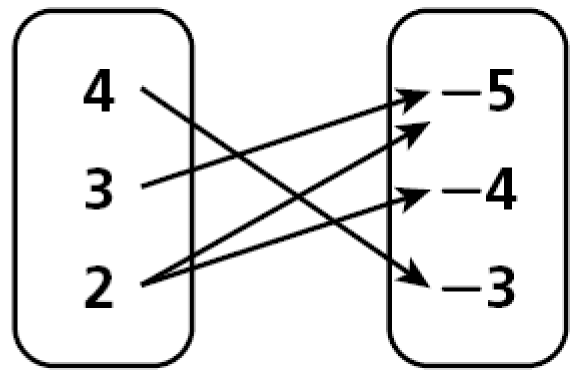
Give the domain and range of each relation. Tell whether the relation is a function and explain.

a. $\{(8, 2), (–4, 1), (–6, 2), (1, 9)\}$

D: $\{–6, –4, 1, 8\}$
R: $\{1, 2, 9\}$

The relation is a function. Each domain value is paired with exactly one range value.

b.

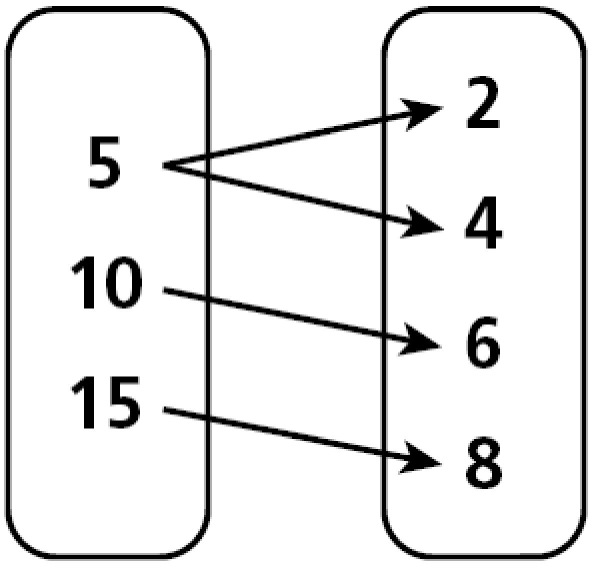


D: $\{2, 3, 4\}$
R: $\{–5, –4, –3\}$

The relation is not a function. The domain value 2 is paired with both –5 and –4.

Your Turn:

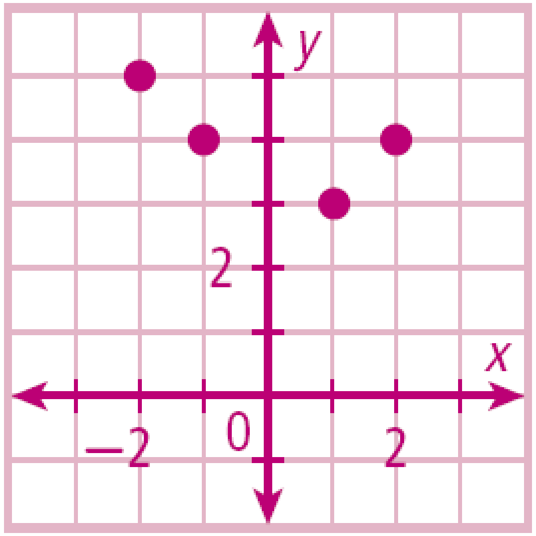
Give the domain and range of the relation. Tell whether the relation is a function. Explain.



D: {5, 10, 15};
R: {2, 4, 6, 8};
The relation is not a function since 5 is paired with 2 and 4.

Your Turn:

Give the domain and range of each relation. Tell whether the relation is a function and explain.



D: {-2, -1, 1, 2}
R: {3, 4, 5}

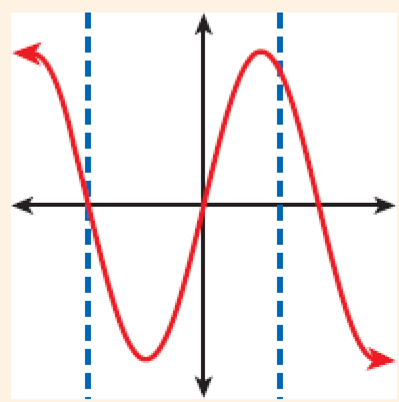
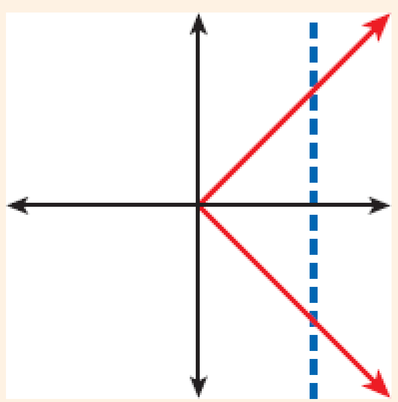
The relation is a function. Each domain value is paired with exactly one range value.

Vertical line Test

- When an equation has two variables, its solutions will be all ordered pairs (x, y) that makes the equation true. Since the solutions are ordered pairs, it is possible to represent them on a graph. When you represent all solutions of an equation on a graph, you are *graphing the equation*.
- Since the solutions of an equation that has two variables are a set of ordered pairs, they are a relation.
- One way to tell if this relation is a function is to graph the equation and use the *vertical-line test*.

Vertical Line Test

The Vertical-Line Test

WORDS	GRAPHS	
Any vertical line will intersect the graph of a function no more than once.	 Function	 Not a function

Example:

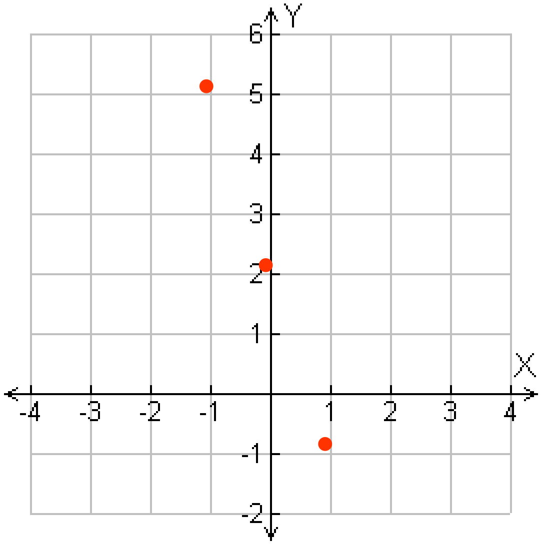
Graph each equation. Then tell whether the equation represents a function.

$-3x + 2 = y$

Step 1 Choose several values of x and generate ordered pairs.

x	$-3x + 2 = y$	(x, y)
-1	$-3(-1) + 2 = 5$	$(-1, 5)$
0	$-3(0) + 2 = 2$	$(0, 2)$
1	$-3(1) + 2 = -1$	$(1, -1)$

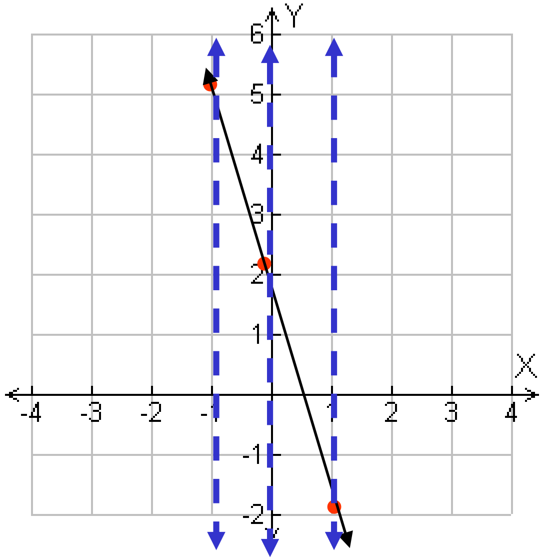
Step 2 Plot enough points to see a pattern.



Example: Continued

Step 3 The points appear to form a line. **Draw a line** through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both “ends” of the line.

Step 4 Use the vertical line test on the graph.



No vertical line will intersect the graph more than once. The equation $-3x + 2 = y$ represents a function.

Example:

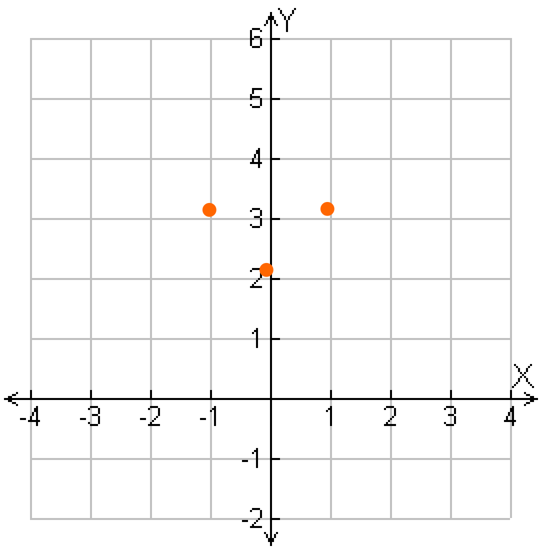
Graph each equation. Then tell whether the equation represents a function.

y **= |*x*| + 2**

Step 1 Choose several values of *x* and generate ordered pairs.

Step 2 Plot enough points to see a pattern.

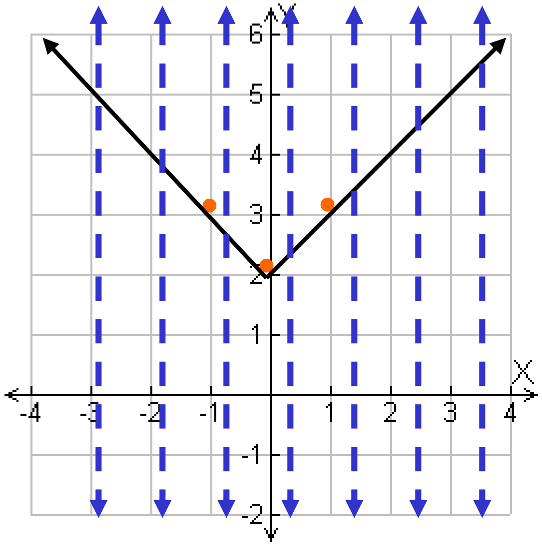
x	**|*x*| + 2 = *y***	**(*x*, *y*)**
–1	**1 + 2 = 3**	**(–1, 3)**
0	**0 + 2 = 2**	**(0, 2)**
1	**1 + 2 = 3**	**(1, 3)**



Example: Continued

Step 3 The points appear to form a V-shaped graph. **Draw two rays** from (0, 2) to show all the ordered pairs that satisfy the function. Draw arrowheads on the end of each ray.

Step 4 Use the vertical line test on the graph.



No vertical line will intersect the graph more than once. The equation $y = |x| + 2$ represents a function.

Your Turn:

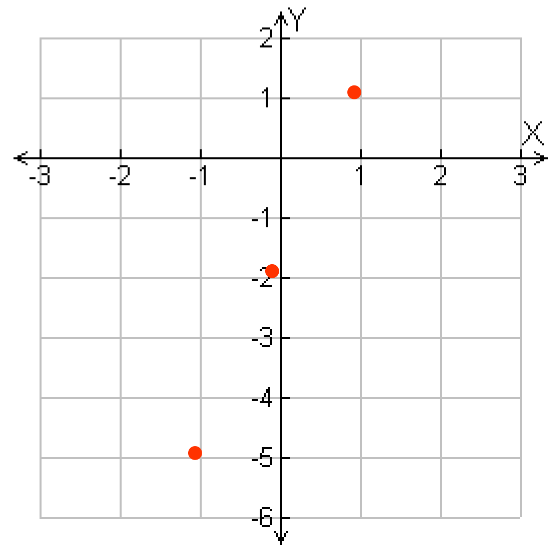
Graph each equation. Then tell whether the equation represents a function.

$y = 3x - 2$

Step 1 Choose several values of *x* and generate ordered pairs.

x	$3x - 2 = y$	(x, y)
-1	$3(-1) - 2 = -5$	$(-1, -5)$
0	$3(0) - 2 = -2$	$(0, -2)$
1	$3(1) - 2 = 1$	$(1, 1)$

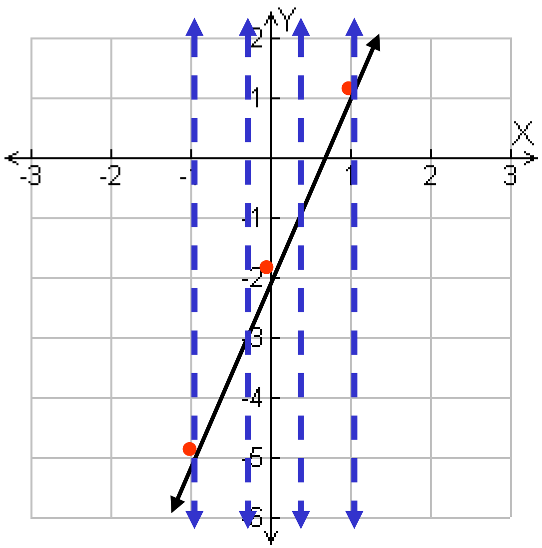
Step 2 Plot enough points to see a pattern.



Your Turn: Continued

Step 3 The points appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both “ends” of the line.

Step 4 Use the vertical line test on the graph.



No vertical line will intersect the graph more than once. The equation $y = 3x - 2$ represents a function.

Your Turn:

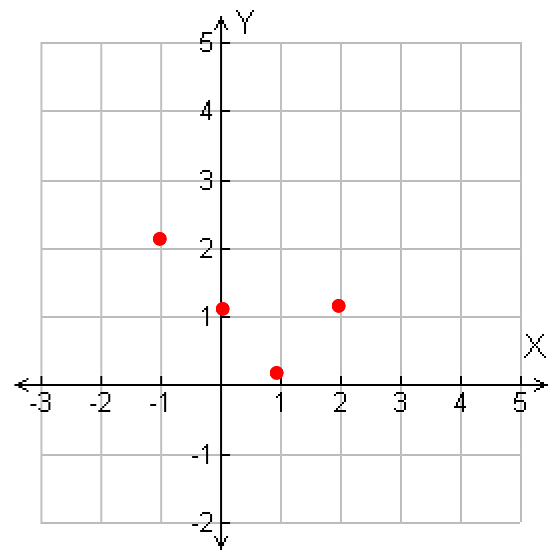
Graph each equation. Then tell whether the equation represents a function.

$y = |x - 1|$

Step 1 Choose several values of *x* and generate ordered pairs.

Step 2 Plot enough points to see a pattern.

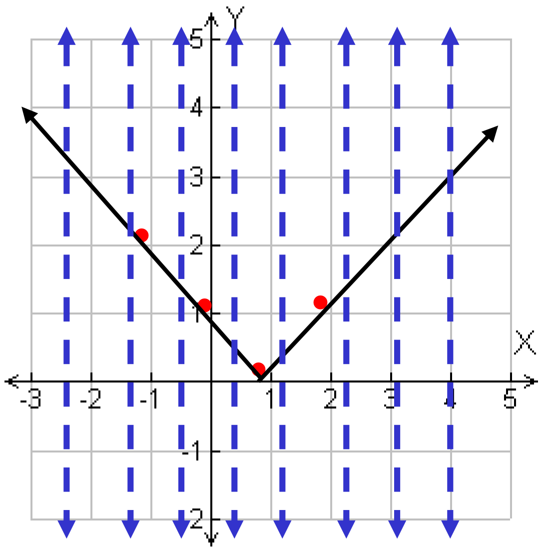
x	$y = \lvert x - 1 \rvert$	(x, y)
–1	**2 = |–1 – 1|**	**(–1, 2)**
0	**1 = |0 – 1|**	**(0, 1)**
1	**0 = |1 – 1|**	**(1, 0)**
2	**1 = |2 – 1|**	**(2, 1)**



Your Turn: Continued

Step 3 The points appear to form a V-shaped graph. **Draw two rays** from (1, 0) to show all the ordered pairs that satisfy the function. Draw arrowheads on the end of each ray.

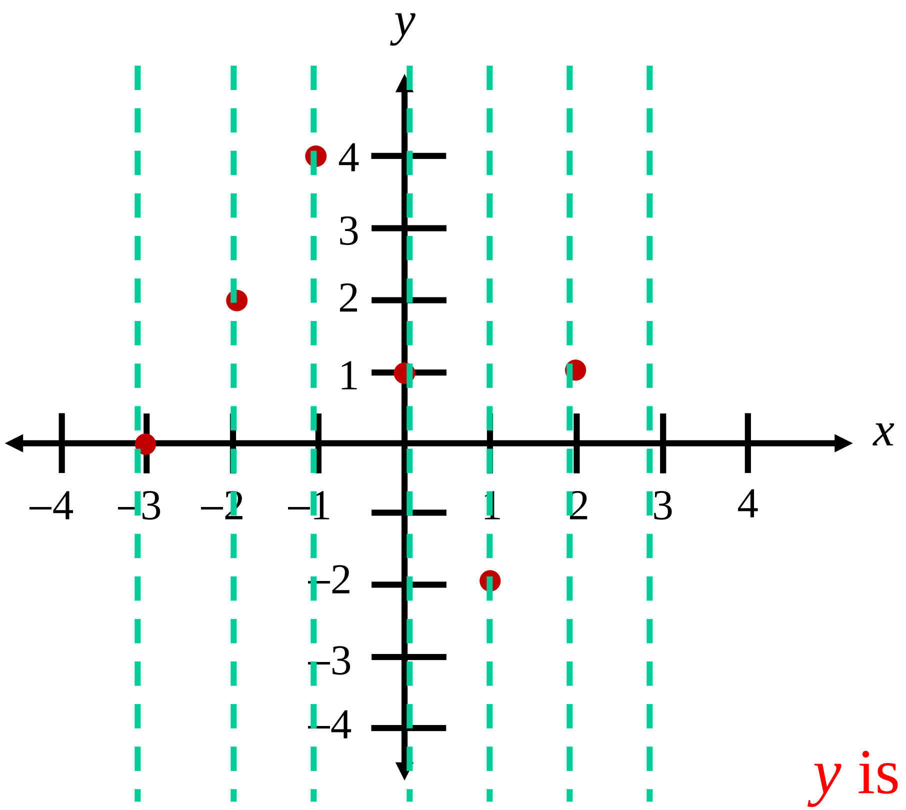
Step 4 Use the vertical line test on the graph.



No vertical line will intersect the graph more than once. The equation $y = |x - 1|$ represents a function.

Example:

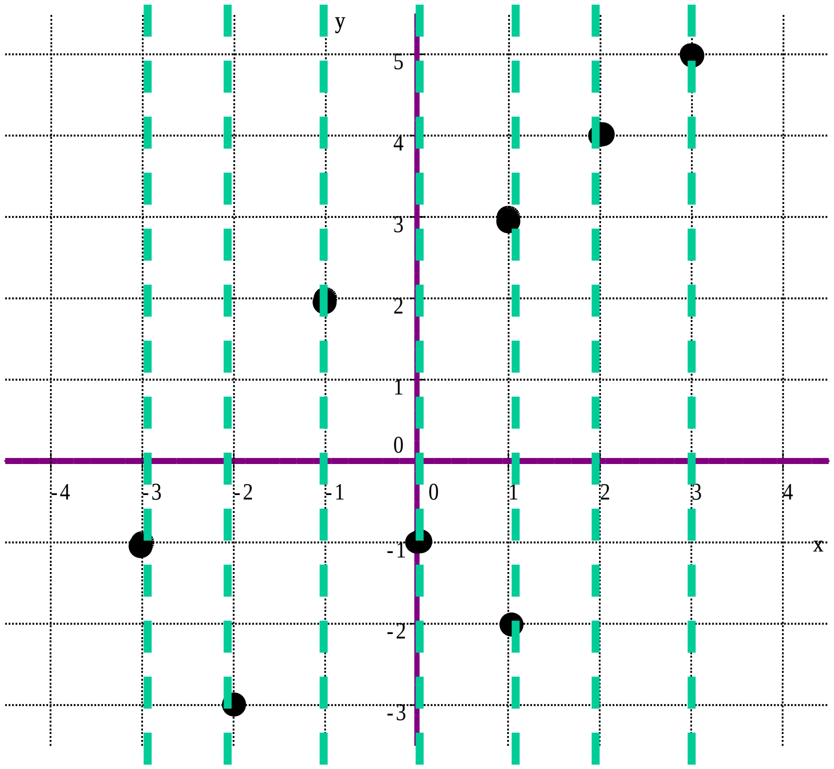
Determine whether the discrete relation is a function.



y is a function of *x*.

Your Turn:

Determine whether the discrete relation is a function.



y is not a function of *x*.

IDENTIFYING FUNCTIONS

Determine if the relationship represents a function.

1.

x	y
2	3
3	4
3	5
2	6

The input x = 2 has two outputs, y = 3 and y = 6. The input x = 3 also has more than one output.

The relationship is not a function.

Determine if the relationship represents a function.

2.

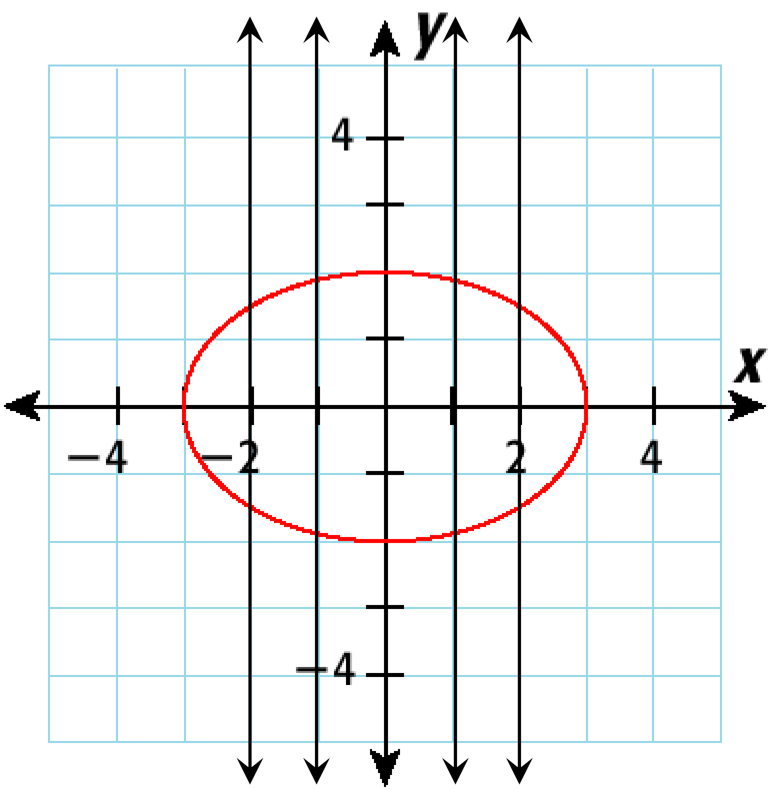
x	y
–1	–1
2	–4
5	– 7
8	–10

Each input has only one output value.

The relationship is a function.

Determine if the relationship represents a function.

3.

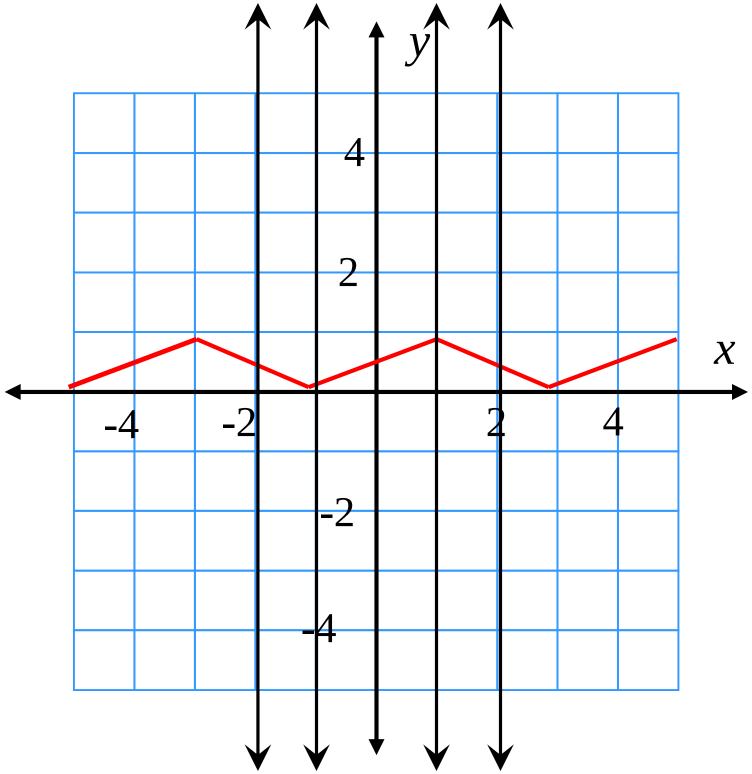


Pass a vertical line across the graph. Many vertical lines intersect the graph at two points.

The relationship is not a function.

Determine if the relationship represents a function.

4.



Pass a vertical line across the graph. No vertical lines intersect the graph at more than one point.

The relationship is a function.

Determine if the relationship represents a function.

5.

x	y
0	0
1	1
2	2
3	3

Each input x has only one output y.

The relationship is a function.

Determine if the relationship represents a function.

6.

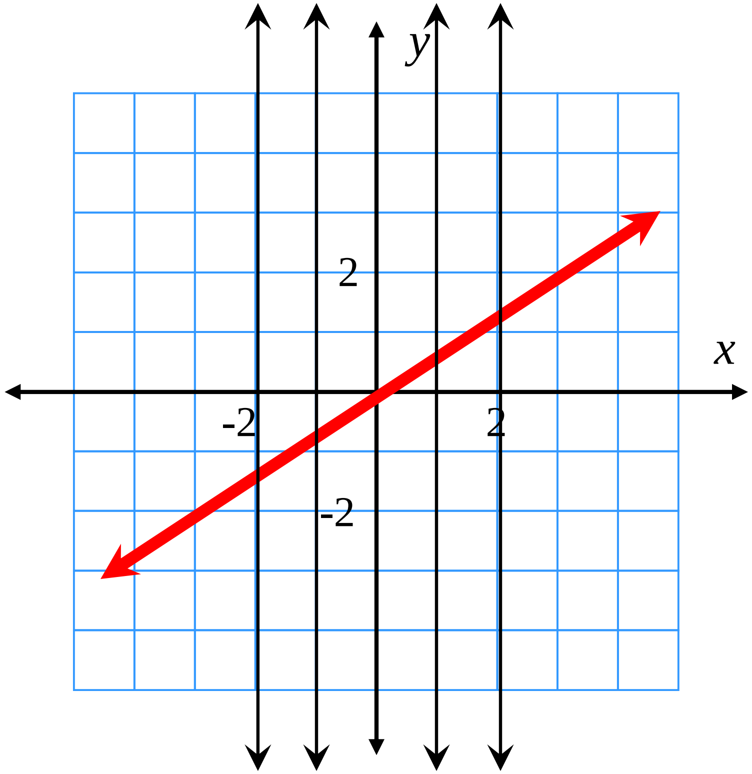
x	*y*
2	1
3	2
4	3
5	4

Each input has only one output value.

The relationship is a function.

Determine if the relationship represents a function.

7.

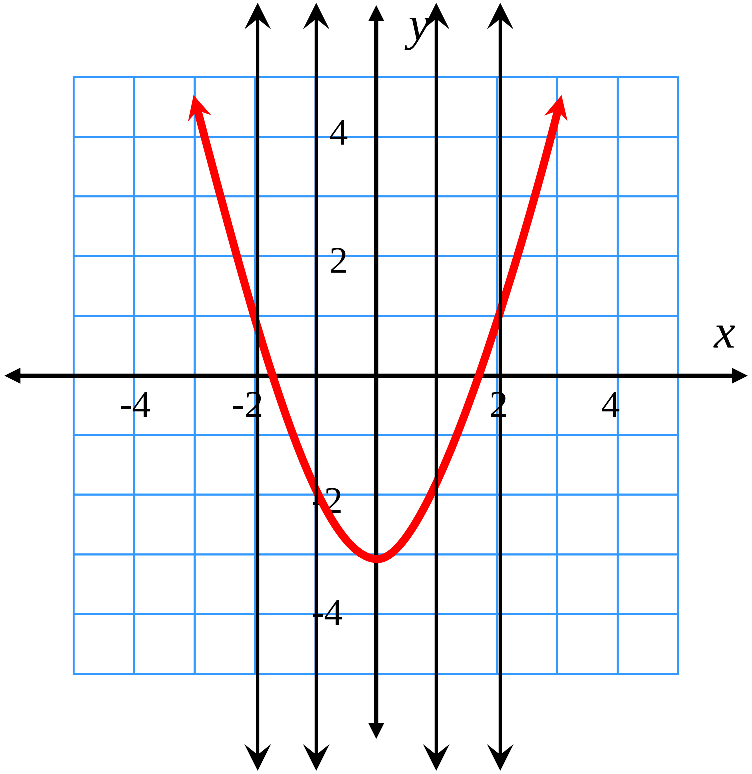


Pass a vertical line across the graph. No vertical lines intersect the graph at more than one point.

The relationship is a function.

Determine if the relationship represents a function.

8.



Pass a vertical line across the graph. No vertical lines intersect the graph at more than one point.

The relationship is a function.

Function Notation

- An algebraic expression that defines a function is a *function rule*.
- If x is the independent variable and y is the dependent variable, then *function notation* for y is $f(x)$, read "f of x," where f names the function.
- When an equation in two variables describes a function, you can use function notation to write it.

Function Notation

The dependent variable is a function of the independent variable.

y is a function of x.

y = f (x)

$$y = f(x)$$

Inputs & Outputs

The *x values*, can be thought of as the inputs and the *y values or* $f(x)$, can be thought of as the outputs.

Function	Input	Output
f	x	$f(x)$
	Independent variable	Dependent variable

Example: Evaluating Functions

Evaluate the function for the given input values.

For $f(x) = 3x + 2$, find $f(x)$ when $x = 7$ and when $x = –4$.

$f(\mathrm{x}) = 3(x) + 2$

$f(7) = 3(7) + 2$

Substitute 7 for x.

$= 21 + 2$

Simplify.

$= 23$

$f(\mathrm{x}) = 3(x) + 2$

$f(–4) = 3(–4) + 2$

Substitute –4 for x.

$= –12 + 2$

Simplify.

$= –10$

Example: Evaluating Functions

Evaluate the function for the given input values.

For $g(t) = 1.5t – 5$, find $g(t)$ when $t = 6$ and when $t = –2$.

$g(t) = 1.5t – 5$

$g(6) = 1.5(6) – 5$

$= 9 – 5$

$= 4$

$g(t) = 1.5t – 5$

$g(–2) = 1.5(–2) – 5$

$= –3 – 5$

$= –8$

Example: Evaluating Functions

Evaluate the function for the given input values.

For $h(r) = \frac{1}{3}r + 2$, find $h(r)$ when $r = 600$ and when $r = -12$.

$$h(r) = \frac{1}{3}r + 2$$

$$h(600) = \frac{1}{3}(600) + 2$$

$$= 200 + 2$$

$$= 202$$

$$h(r) = \frac{1}{3}r + 2$$

$$h(-12) = \frac{1}{3}(-12) + 2$$

$$= -4 + 2$$

$$= -2$$

Your Turn:

Evaluate the function for the given input values.

For $h(c) = 2c - 1$, find $h(c)$ when $c = 1$ and when $c = -3$.

$$h(c) = 2c - 1$$

$$h(1) = 2(1) - 1$$

$$= 2 - 1$$

$$= 1$$

$$h(c) = 2c - 1$$

$$h(-3) = 2(-3) - 1$$

$$= -6 - 1$$

$$= -7$$

Your Turn:

Evaluate each function for the given input values.

For $g(t) = \frac{1}{4}t + 1$ find $g(t)$ when $t = -24$ and when $t = 400$.

$$g(t) = \frac{1}{4}t + 1$$

$$g(-24) = \frac{1}{4}(-24) + 1$$

$$= -6 + 1$$

$$= -5$$

$$g(t) = \frac{1}{4}t + 1$$

$$g(400) = \frac{1}{4}(400) + 1$$

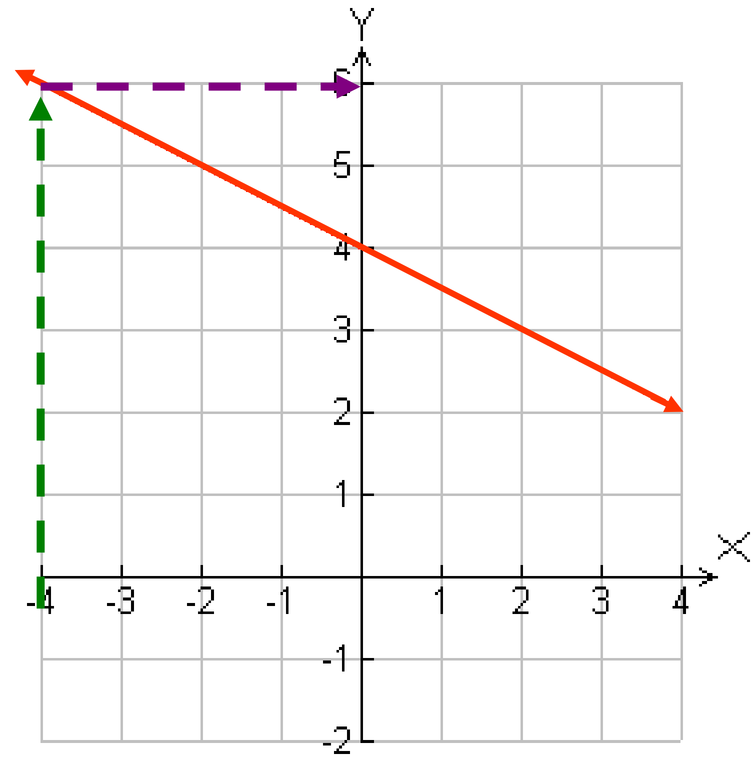
$$= 100 + 1$$

$$= 101$$

Example: Evaluating Functions from a Graph

Use a graph of the function $f(x) = -\frac{1}{2}x + 4$ to find the value of $f(x)$ when $x = -4$.

Locate –4 on the *x*-axis. Move up to the graph of the function. Then move right to the *y*-axis to find the corresponding value of *y*.

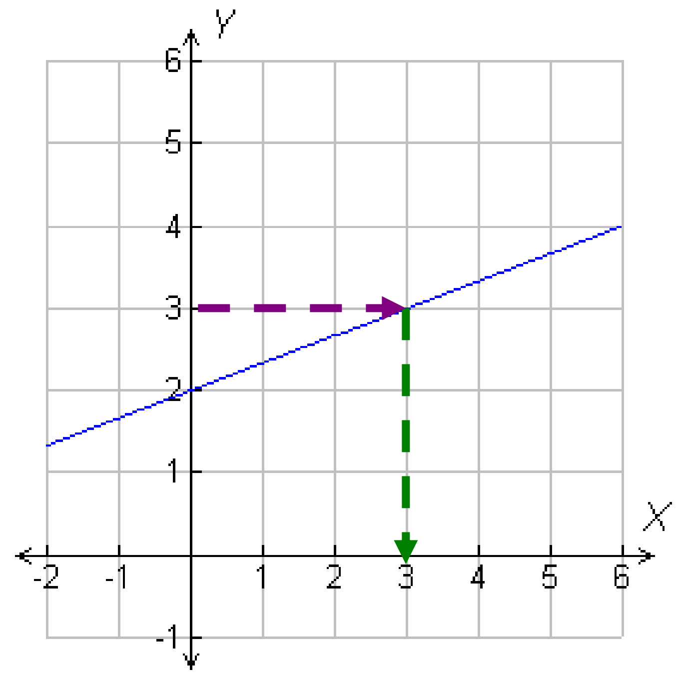


$f(-4) = 6$

Your Turn:

Use the graph of $f(x) = \frac{1}{3}x + 2$ to find the value of x when $f(x) = 3$.

Locate 3 on the *y*-axis. Move right to the graph of the function. Then move down to the *x*-axis to find the corresponding value of *x*.



f(3) = 3

Assignment

- Worksheet