

## Section 4-1 Congruent Figures

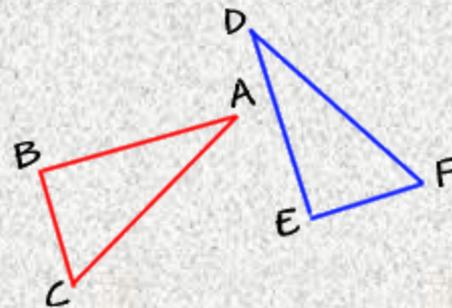
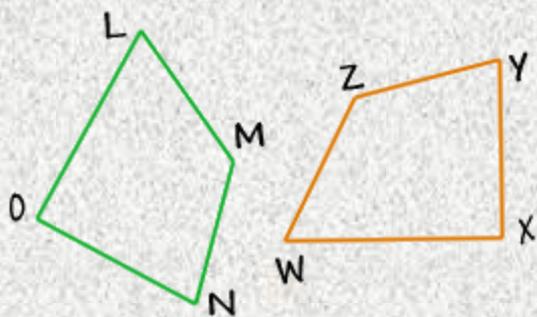
### Objectives:

- recognize congruent figures and their corresponding parts

**Congruent figures:**

**Have the same size and shape**

**Shapes may have a different orientation**



# Congruent Polygons

Congruent Polygons have congruent corresponding parts

- Congruent sides
- Congruent Angles
- Matching vertices are corresponding vertices
  - always list corresponding parts in the same order

Line  $AB \cong$  line  $DE$

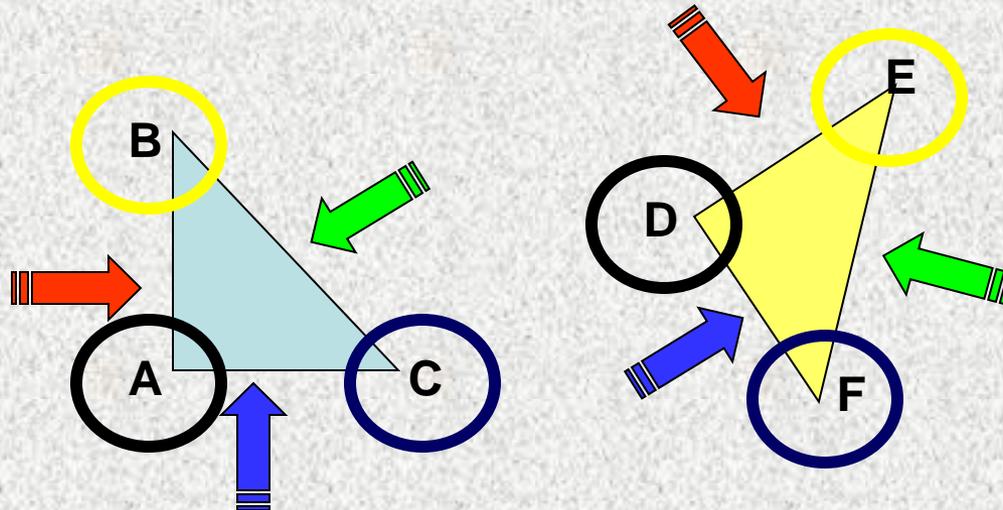
Line  $BC \cong$  line  $EF$

Line  $CA \cong$  line  $FD$

$A \cong D$

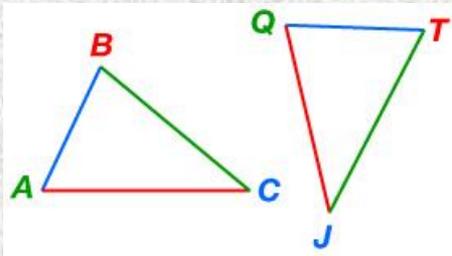
$B \cong E$

$C \cong F$



## Example

$\triangle ABC \cong \triangle QTJ$ . List the congruent corresponding parts.



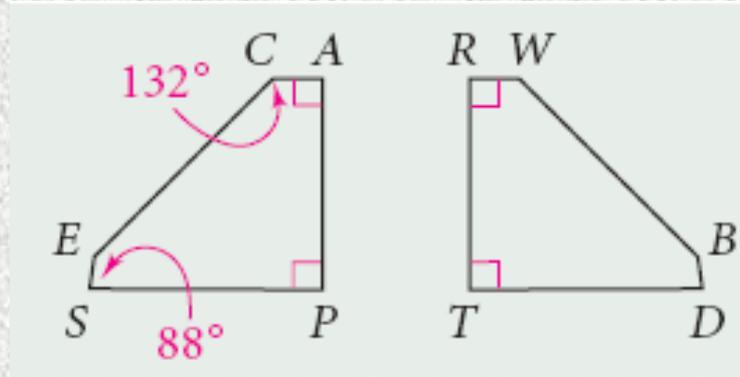
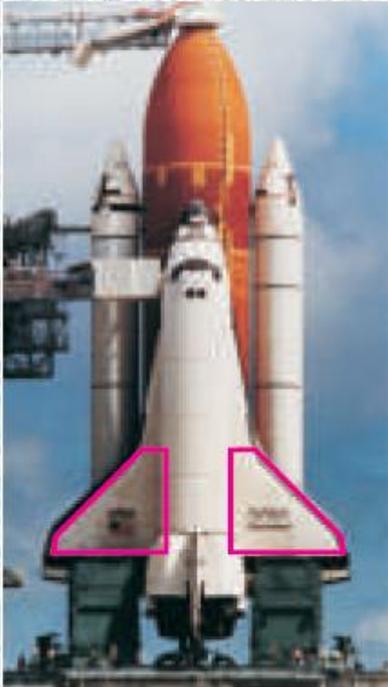
List the corresponding vertices in the same order.

Angles:  $\angle A \cong \angle Q$     $\angle B \cong \angle T$     $\angle C \cong \angle J$

List the corresponding sides in the same order.

Sides:  $\overline{AB} \cong \overline{QT}$     $\overline{BC} \cong \overline{TJ}$     $\overline{AC} \cong \overline{QJ}$

The fins of the space shuttle suggest congruent pentagons. Find the  $m\angle B$ .



What is the angle corresponding to B?

Use the Polygon Angle Sum Thm:  
 $(n - 2)180 = (5 - 2)180 = 540$

$\triangle XYZ \cong \triangle KLM$ ,  $m\angle Y = 67$ , and  $m\angle M = 48$ . Find  $m\angle X$ .

Use the Triangle Angle-Sum Theorem and the definition of congruent polygons to find  $m\angle X$ .

$$m\angle X + m\angle Y + m\angle Z = 180$$

$$m\angle Z = m\angle M$$

$$m\angle Z = 48$$

$$m\angle X + 67 + 48 = 180$$

$$m\angle X + 115 = 180$$

$$m\angle X = 65$$

Triangle Angle-Sum Theorem

Corresponding angles of congruent triangles that are congruent

Substitute 48 for  $m\angle M$ .

Substitute.

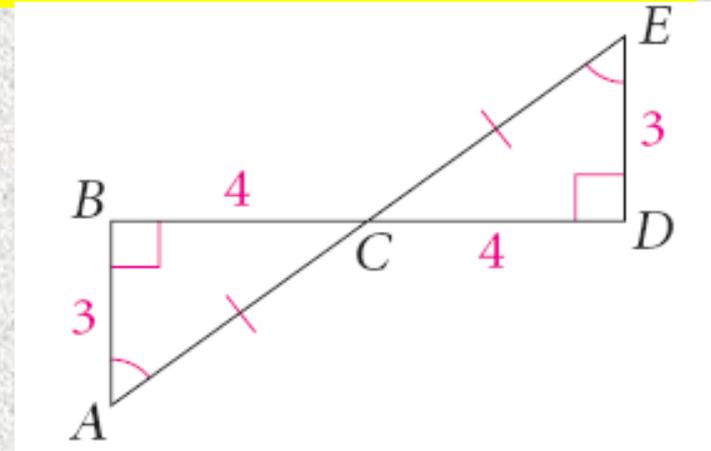
Simplify.

Subtract 115 from each side.

## Are the Two Triangles Congruent?

Two triangles are congruent if they have:

- 3 pairs of  $\cong$  corresponding sides
- 3 pairs of  $\cong$  corresponding angles



Look at the diagram. What do you know by the markings?

$$\overline{AC} \cong \overline{EC} \quad \text{Given}$$

$$\overline{AB} \cong \overline{ED} \quad \therefore \overline{AB} = 3 = \overline{ED}$$

$$\overline{BC} \cong \overline{DC} \quad \therefore \overline{BC} = 4 = \overline{DC}$$

$$\angle A \cong \angle E \quad \text{Given}$$

$$\angle B \cong \angle D \quad \text{Right } \angle\text{s are } \cong$$

What is true about angles ACB and DCE? Explain.

They are vertical angles, so they are congruent.

# Proving Triangle Congruency

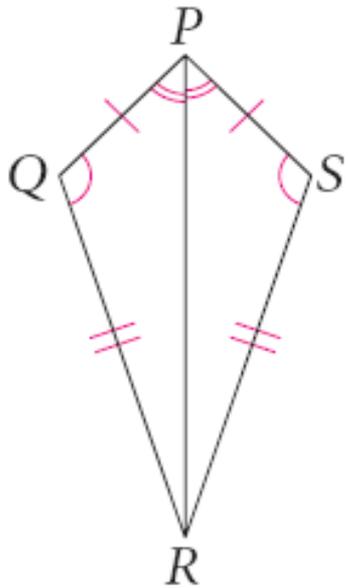
## Theorem 4-1

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

$$\angle C \cong \angle F$$



Use the information in the diagram to give a reason why each statement is true.

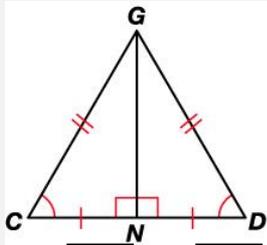


Statement	Reason
$\overline{PQ} \cong \overline{PS}, \overline{QR} \cong \overline{SR}$	<b>Given</b>
$\overline{PR} \cong \overline{PR}$	<b>Reflexive Prop of <math>\cong</math></b>
$\angle Q \cong \angle S, \angle QPR \cong \angle SPR$	<b>Given</b>
$\angle QRP \cong \angle SRP$	<b>Theorem 4-1</b>
$\triangle PQR \cong \triangle PSR$	<b>Def of <math>\cong</math> triangles</b>

## Examples

Given:  $\overline{CG} \cong \overline{DG}$ ,  $\overline{CN} \cong \overline{DN}$ ,  $\angle C \cong \angle D$ ,  $\angle CNG$  and  $\angle DNG$  are right angles.

Prove:  $\triangle CNG \cong \triangle DNG$ .



Congruent triangles have three congruent corresponding sides and three congruent corresponding angles.

Examine the diagram, and list the congruent corresponding parts for  $\triangle CNG$  and  $\triangle DNG$ .

a.  $\overline{CG} \cong \overline{DG}$

Given

b.  $\overline{CN} \cong \overline{DN}$

Given

c.  $\overline{GN} \cong \overline{GN}$

Reflexive Property of Congruence

d.  $\angle C \cong \angle D$

Given

e.  $\angle CNG \cong \angle DNG$

Right angles are congruent.

f.  $\angle CGN \cong \angle DGN$

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent. (Theorem 4-1.)

g.  $\triangle CNG \cong \triangle DNG$

Definition of  $\cong$  triangles

**Practice!**

**Pg. 182-184**

**# 1-31 odd**

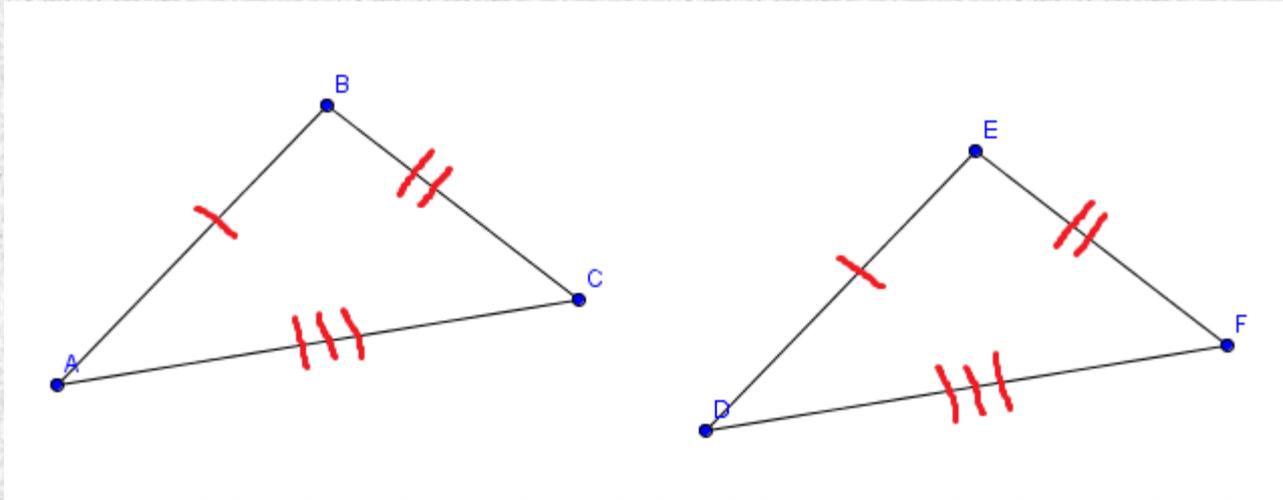
**#44 turn in for 5 extra credit points!!**

# Proving Triangles Congruent: SSS and SAS

Section 4-2

# Side-Side-Side (SSS) Congruence Postulate

If 3 sides of one triangle are congruent to 3 sides of another triangle, then the two triangles are congruent.

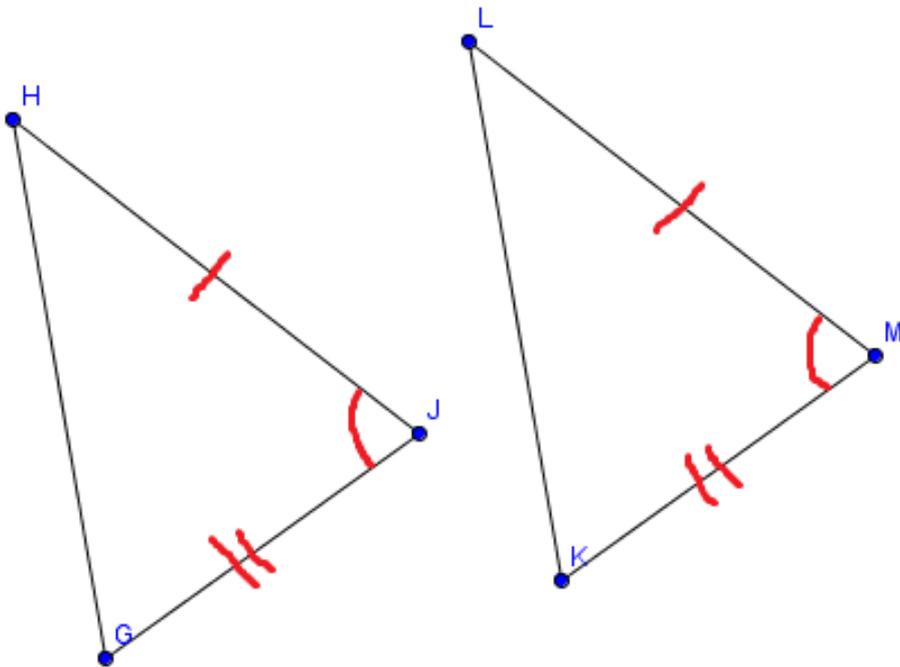


If all 3 sides match, then  $\triangle ABC \cong \triangle DEF$

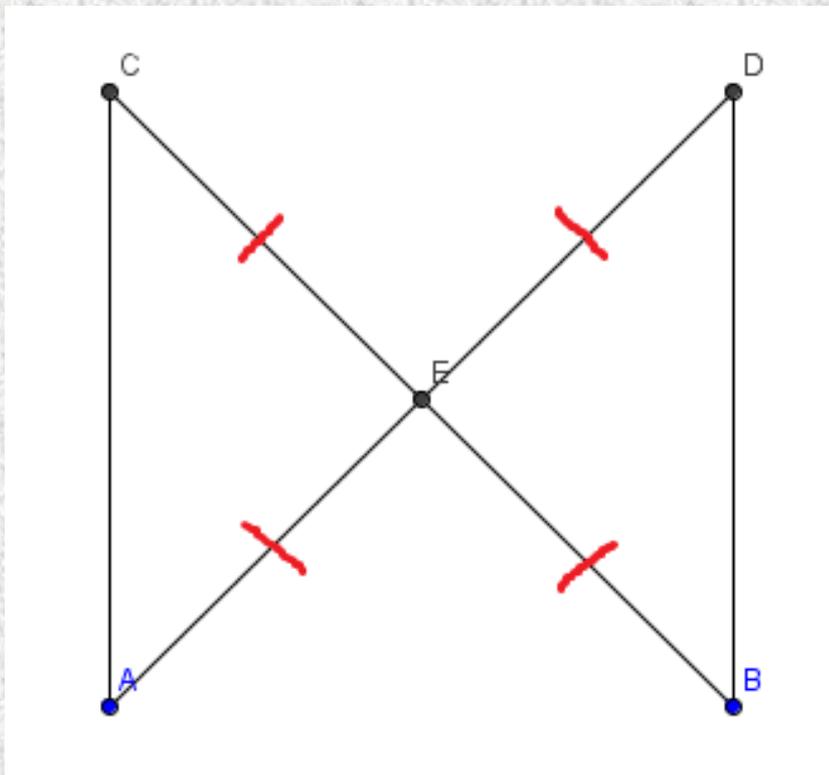
# Side-Angle-Side (SAS) Congruence Postulate

If 2 sides of one triangle **and the angle they form** are congruent to 2 sides of another triangle and the

**angle they form, then the triangles are congruent.**



**Decide if the triangles are congruent. If so, state the congruence postulate that applies.**



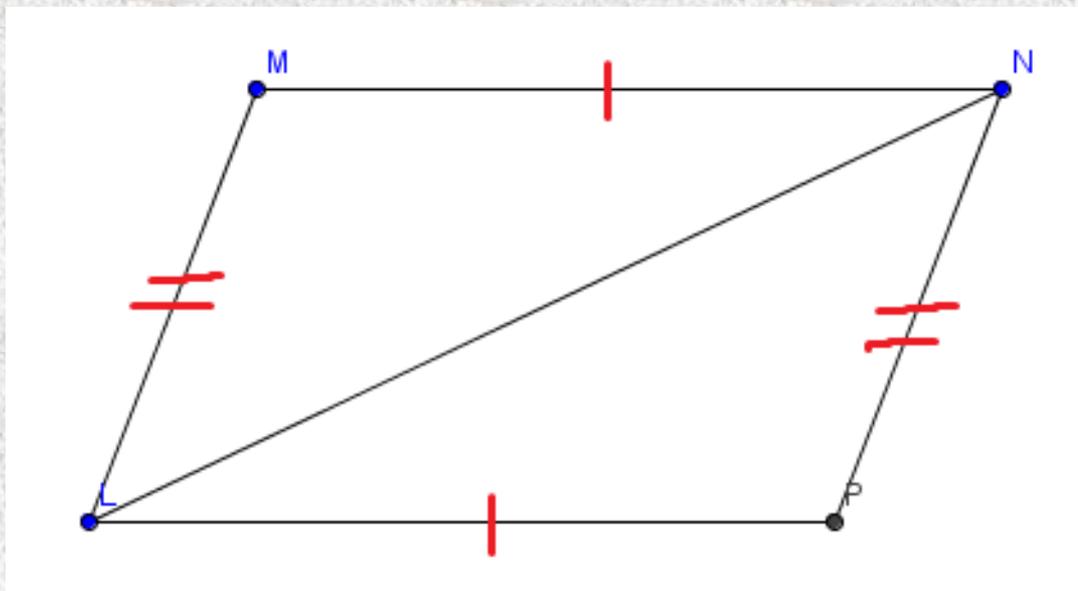
$\angle CEA$  and  $\angle DEB$  are vertical angles.

$$\overline{CE} \cong \overline{DE}; \overline{AE} \cong \overline{BE}$$

$\triangle CEA$  and  $\triangle DEB$  are congruent!

**SAS**

**Decide if the triangles are congruent. If so, state the congruence postulate that applies.**



$\overline{LN}$  is shared by both triangles.

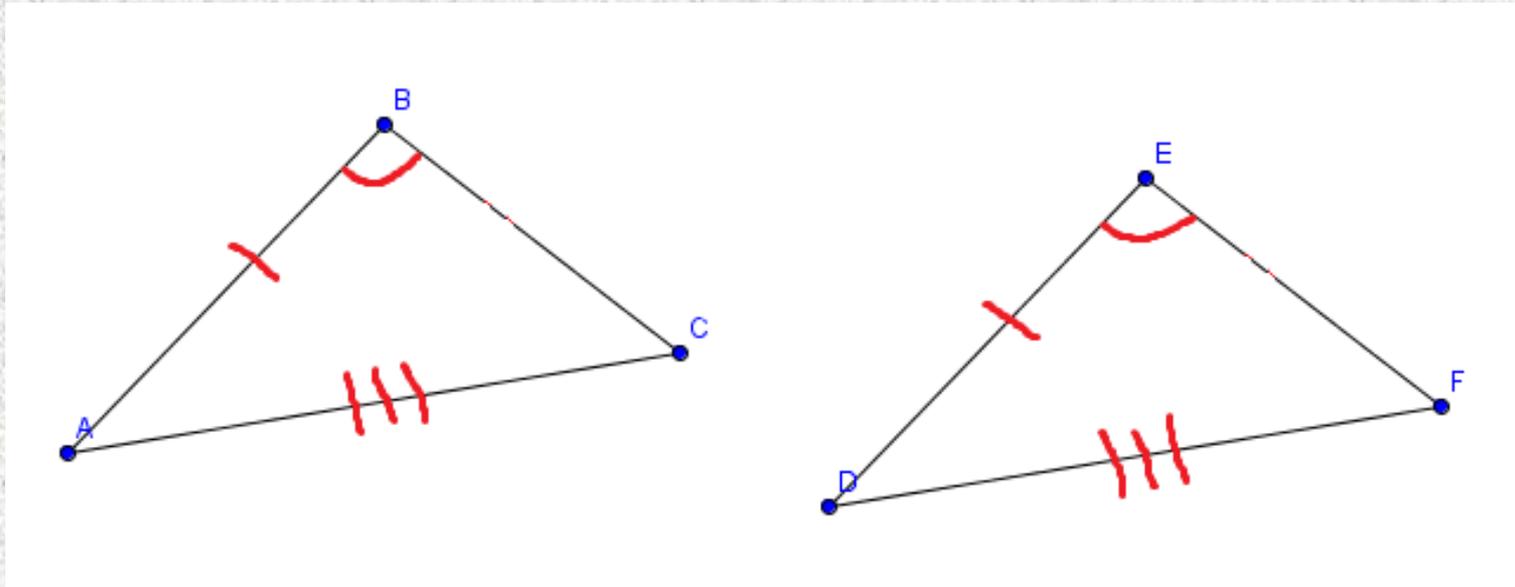
$$\overline{LN} \cong \overline{LN}$$

$$\overline{MN} \cong \overline{PL}; \overline{NP} \cong \overline{LM}$$

$\triangle LMN$  and  $\triangle NPL$  are congruent!

SSS

**Decide if the triangles are congruent. If so, state the congruence postulate that applies.**



***NO!!!***

# Assignment

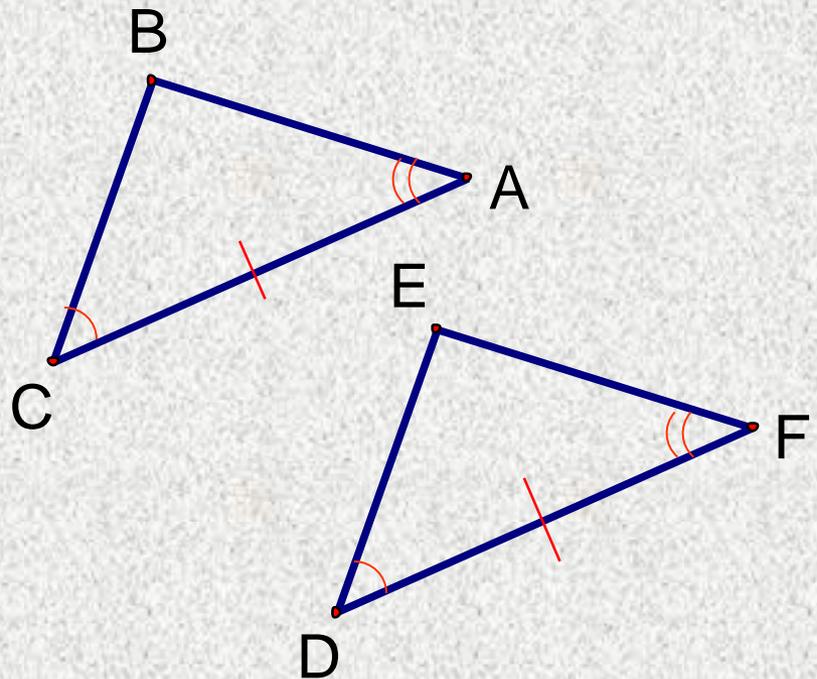
Pg 189-191 #1-39 odd

# Section 4-3

## Triangle congruence by ASA and AAS

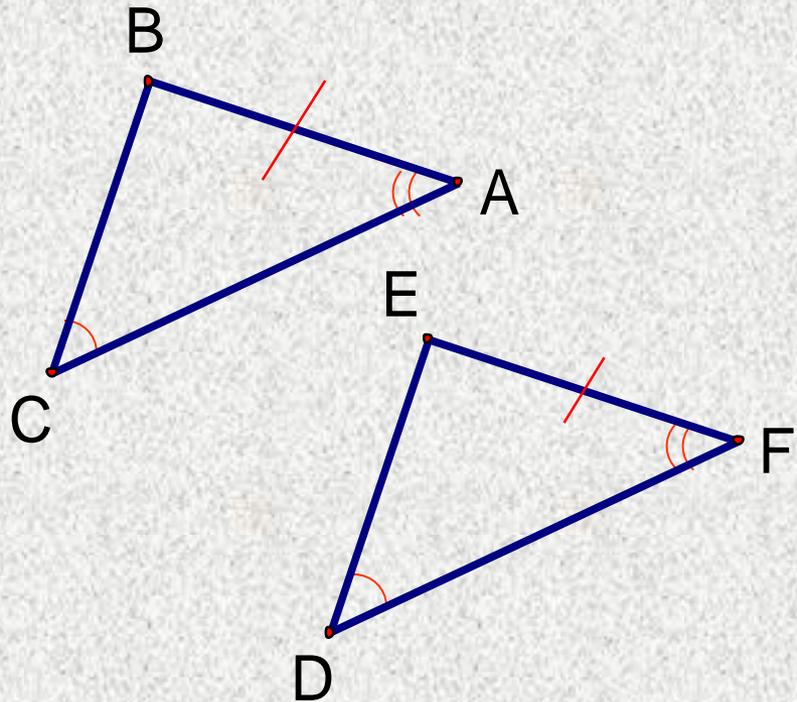
# Postulate 4-3: Angle-Side-Angle (ASA) Congruence Postulate

- If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent.



# Theorem 4-2: Angle-Angle-Side (AAS) Congruence Theorem

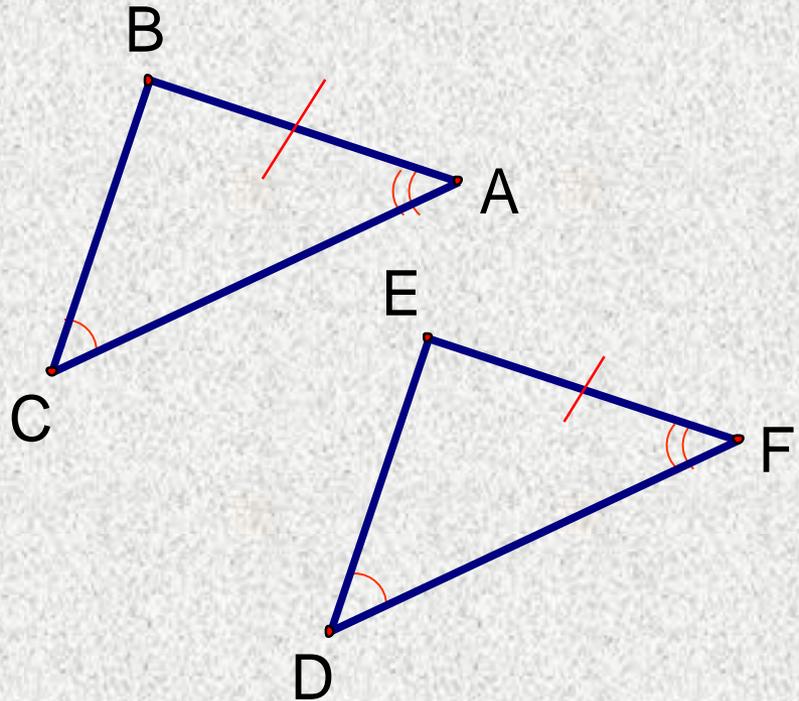
- If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the triangles are congruent.



# Theorem 4-2: Angle-Angle-Side (AAS) Congruence Theorem

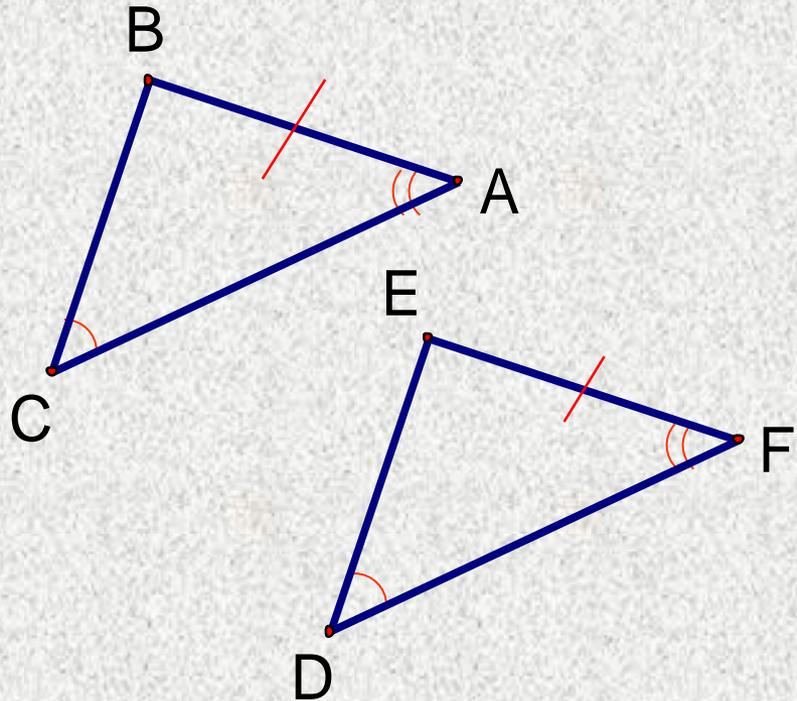
Given:  $\angle A \cong \angle D$ ,  $\angle C \cong \angle F$ ,  $BC \cong EF$

Prove:  $\triangle ABC \cong \triangle DEF$



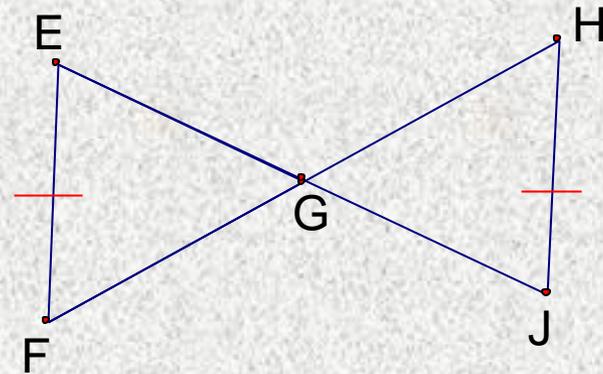
# Theorem 4-2: Angle-Angle-Side (AAS) Congruence Theorem

You are given that two angles of  $\triangle ABC$  are congruent to two angles of  $\triangle DEF$ . By the Third Angles Theorem, the third angles are also congruent. That is,  $\angle B \cong \angle E$ . Notice that  $BC$  is the side included between  $\angle B$  and  $\angle C$ , and  $EF$  is the side included between  $\angle E$  and  $\angle F$ . You can apply the ASA Congruence Postulate to conclude that  $\triangle ABC \cong \triangle DEF$ .



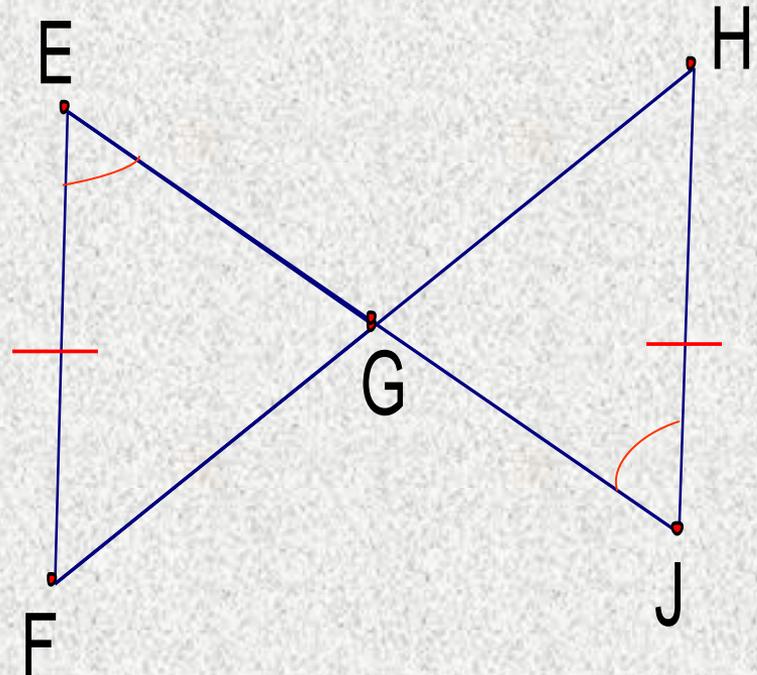
# Ex. 1 Developing Proof

Is it possible to prove the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.



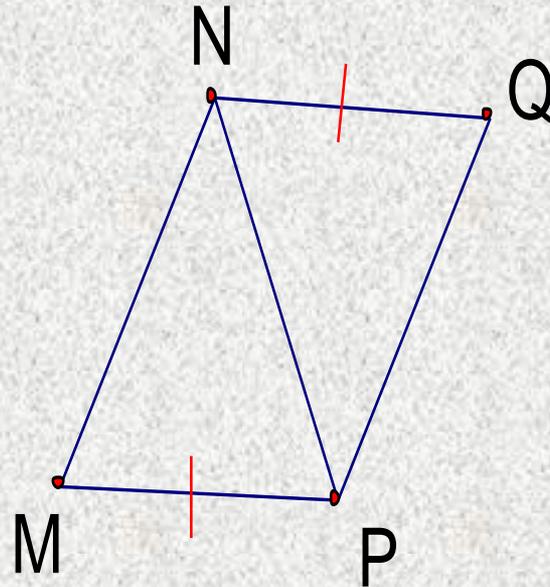
# Ex. 1 Developing Proof

A. In addition to the angles and segments that are marked,  $\angle EGF \cong \angle JGH$  by the Vertical Angles Theorem. Two pairs of corresponding angles and one pair of corresponding sides are congruent. You can use the AAS Congruence Theorem to prove that  $\triangle EFG \cong \triangle JHG$ .



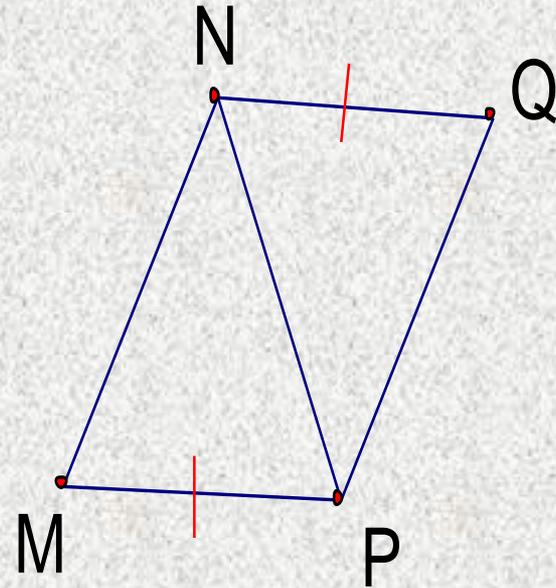
# Ex. 1 Developing Proof

Is it possible to prove the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.



# Ex. 1 Developing Proof

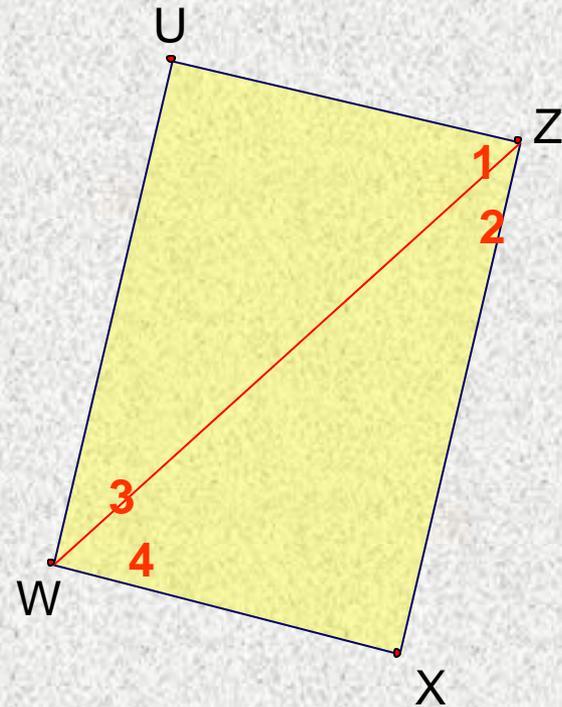
B. In addition to the congruent segments that are marked,  $NP \cong NP$ . Two pairs of corresponding sides are congruent. This is not enough information to prove the triangles are congruent.



# Ex. 1 Developing Proof

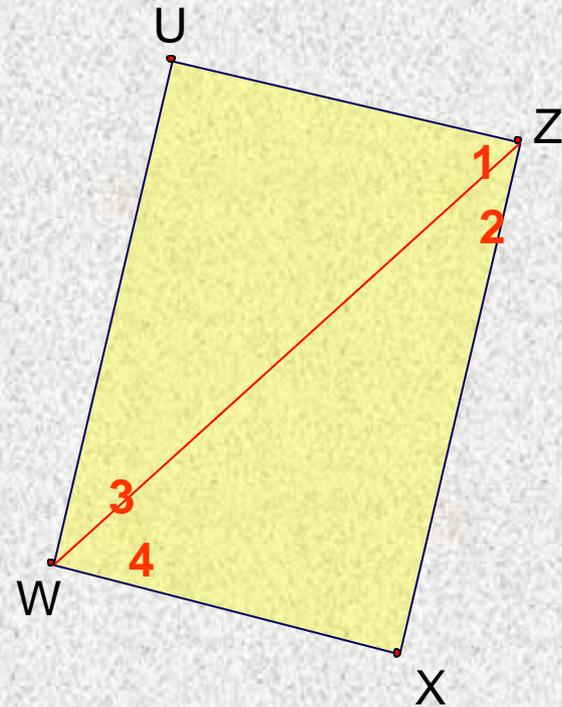
Is it possible to prove the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.

$UZ \parallel WX$  AND  $UW \parallel WX$ .



# Ex. 1 Developing Proof

The two pairs of parallel sides can be used to show  $\angle 1 \cong \angle 3$  and  $\angle 2 \cong \angle 4$ . Because the included side  $WZ$  is congruent to itself,  $\triangle WUZ \cong \triangle ZXW$  by the ASA Congruence Postulate.



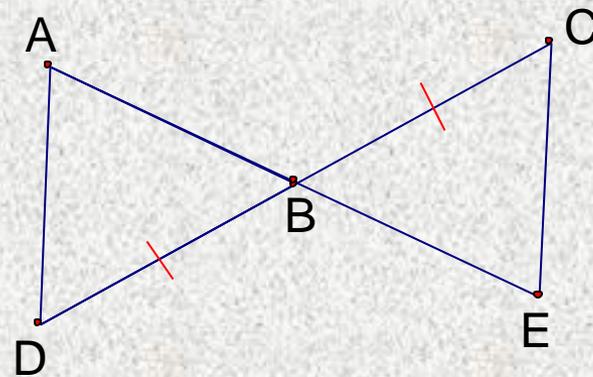
# Ex. 2 Proving Triangles are Congruent

Given:  $AD \parallel EC$ ,  $BD \cong BC$

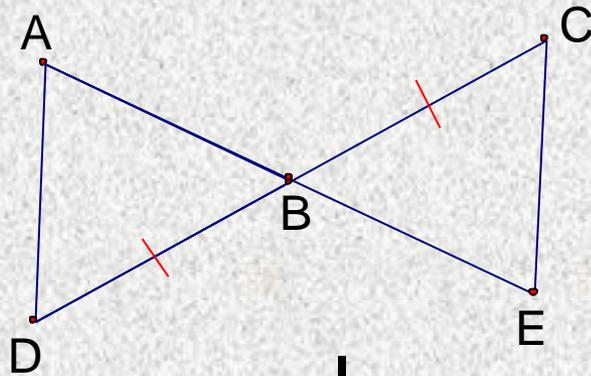
Prove:  $\triangle ABD \cong \triangle EBC$

Plan for proof: Notice that  $\angle ABD$  and  $\angle EBC$  are congruent. You are given that  $BD \cong BC$

. Use the fact that  $AD \parallel EC$  to identify a pair of congruent angles.



# Proof:



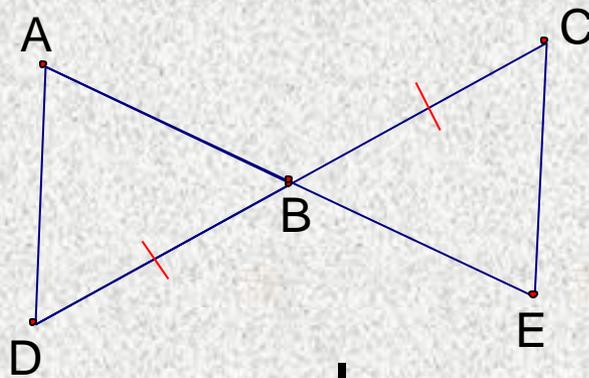
Statements:

Reasons:

1.  $BD \cong BC$
2.  $AD \parallel EC$
3.  $\angle D \cong \angle C$
4.  $\angle ABD \cong \angle EBC$
5.  $\triangle ABD \cong \triangle EBC$

1.

# Proof:



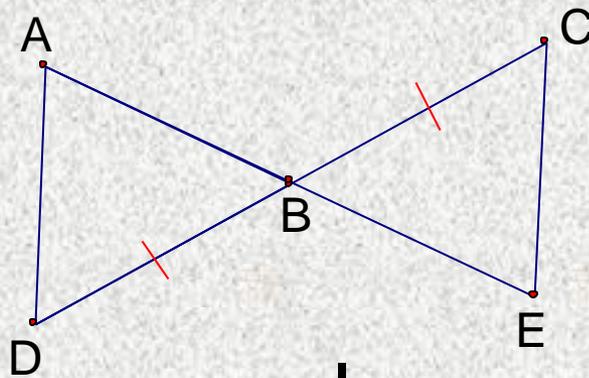
Statements:

1.  $BD \cong BC$
2.  $AD \parallel EC$
3.  $\angle D \cong \angle C$
4.  $\angle ABD \cong \angle EBC$
5.  $\triangle ABD \cong \triangle EBC$

Reasons:

1. Given

# Proof:



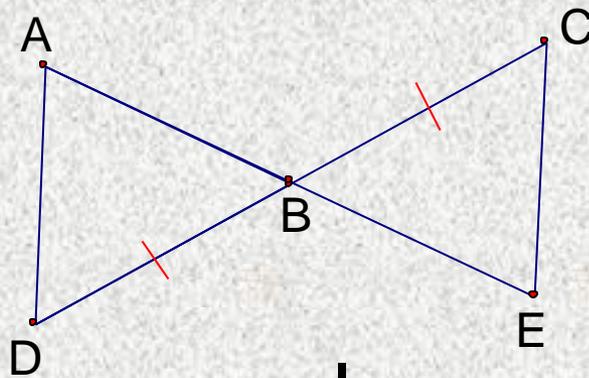
Statements:

1.  $BD \cong BC$
2.  $AD \parallel EC$
3.  $\angle D \cong \angle C$
4.  $\angle ABD \cong \angle EBC$
5.  $\triangle ABD \cong \triangle EBC$

Reasons:

1. Given
2. Given

# Proof:



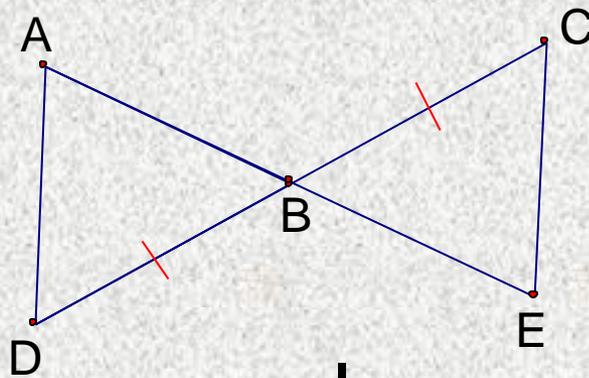
Statements:

1.  $BD \cong BC$
2.  $AD \parallel EC$
3.  $\angle D \cong \angle C$
4.  $\angle ABD \cong \angle EBC$
5.  $\triangle ABD \cong \triangle EBC$

Reasons:

1. Given
2. Given
3. Alternate Interior Angles

# Proof:



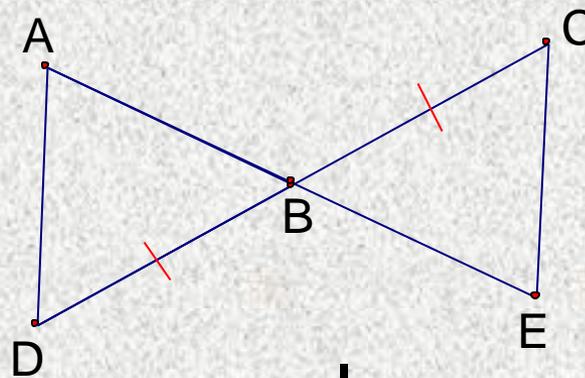
Statements:

1.  $BD \cong BC$
2.  $AD \parallel EC$
3.  $\angle D \cong \angle C$
4.  $\angle ABD \cong \angle EBC$
5.  $\triangle ABD \cong \triangle EBC$

Reasons:

1. Given
2. Given
3. Alternate Interior Angles
4. Vertical Angles Theorem

# Proof:



## Statements:

1.  $BD \cong BC$
2.  $AD \parallel EC$
3.  $\angle D \cong \angle C$
4.  $\angle ABD \cong \angle EBC$
5.  $\triangle ABD \cong \triangle EBC$

## Reasons:

1. Given
2. Given
3. Alternate Interior Angles
4. Vertical Angles Theorem
5. ASA Congruence Theorem

# Note:

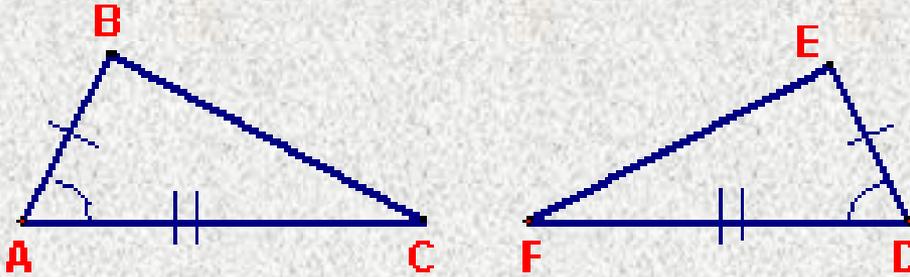
- You can often use more than one method to prove a statement. In Example 2, you can use the parallel segments to show that  $\angle D \cong \angle C$  and  $\angle A \cong \angle E$ . Then you can use the AAS Congruence Theorem to prove that the triangles are congruent.

Practice Pg. 197-200 # 1-41  
odd

# Using congruent Triangles: CPCTC

Section 4-4

# Match the Corresponding Parts



Given:  $\overline{AB} \cong \overline{DE}$

$\overline{AC} \cong \overline{DF}$

$\angle A \cong \angle D$

Therefore:  $\triangle BAC \cong \triangle EDF$

$\angle A$

DF

$\angle B$

DE

$\angle C$

$\angle D$

AB

EF

BC

$\angle E$

AC

$\angle F$

# C.P.C.T.C.

- **C**orresponding
- **P**arts (of)
- **C**ongruent
- **T**riangles (are)
- **C**ongruent

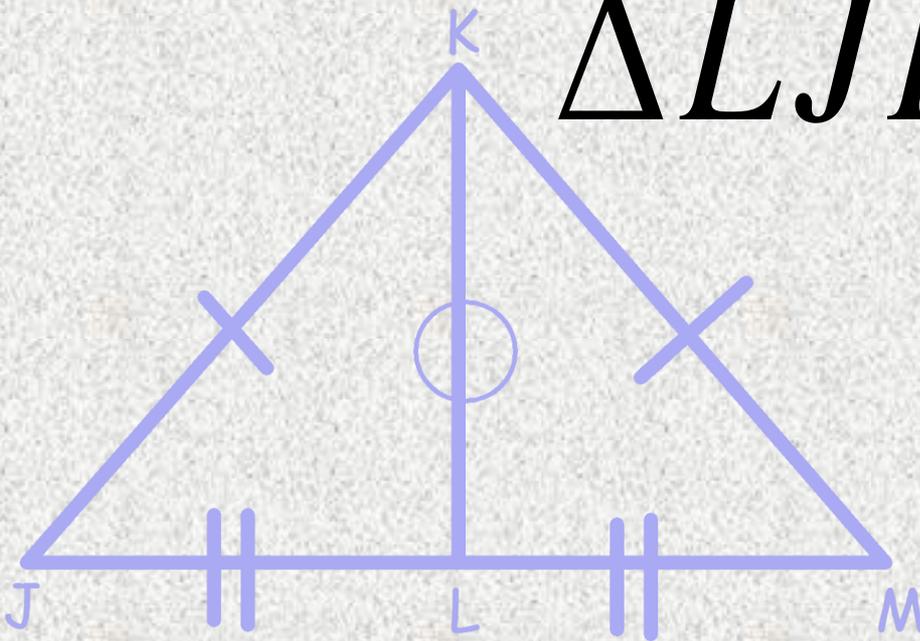
**If two triangles are congruent, then their corresponding parts are also congruent.**

**Very important!!!!**

- Before you use CPCTC you must prove or know that the two triangles congruent!!!

# Using CPCTC

$$\triangle LJK \cong \triangle LMK$$

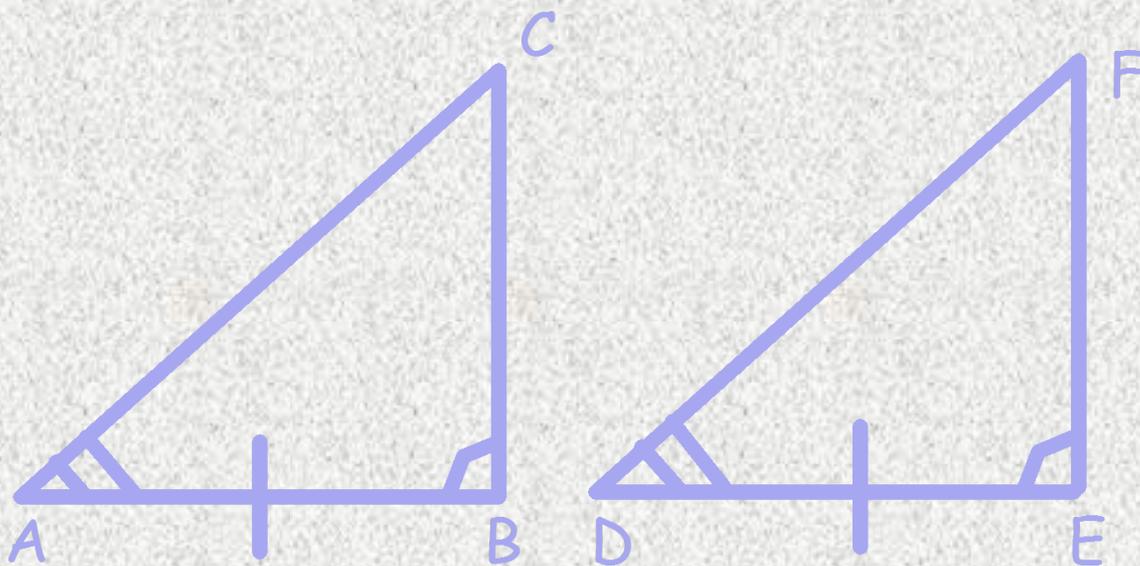


$$\overline{JK} \cong \overline{MK}, \overline{JL} \cong \overline{ML}, \overline{KL} \cong \overline{KL}$$

$$\angle JLK \cong \angle MLK, \angle LJK \cong \angle LMK, \angle J \cong \angle M$$

# Using CPCTC

$$\triangle ABC \cong \triangle DEF$$

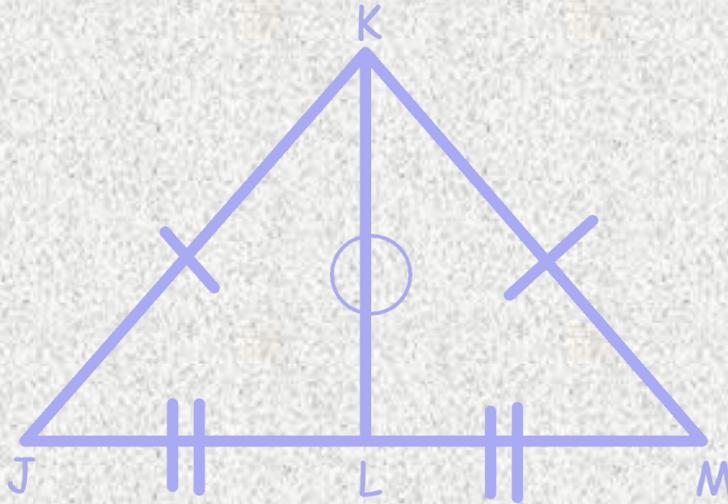


**With your partner  
write down all  
congruent parts.**

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

$$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$$

# Find the value of all the angles



$$\Delta JKL \cong \Delta MKL$$

$$\angle J = 3x + 2$$

$$\angle JLK = 90^\circ$$

$$\angle M = 5x - 32$$

$$\angle J = 53$$

$$\angle JKL = 37$$

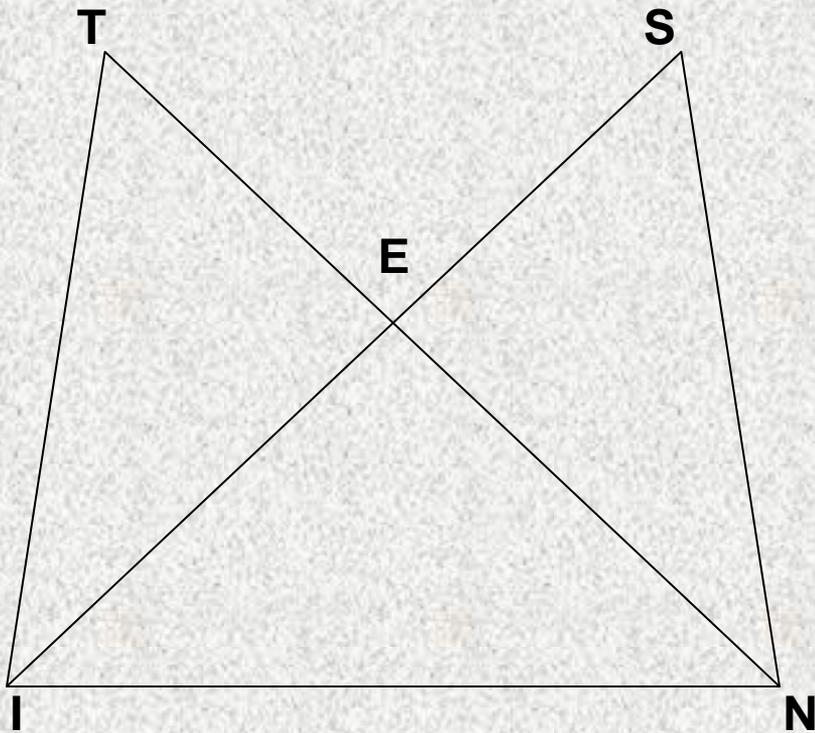
$$\angle JLK = 90$$

$$\angle MLK = 90$$

$$\angle MKL = 37$$

$$\angle M = 53$$

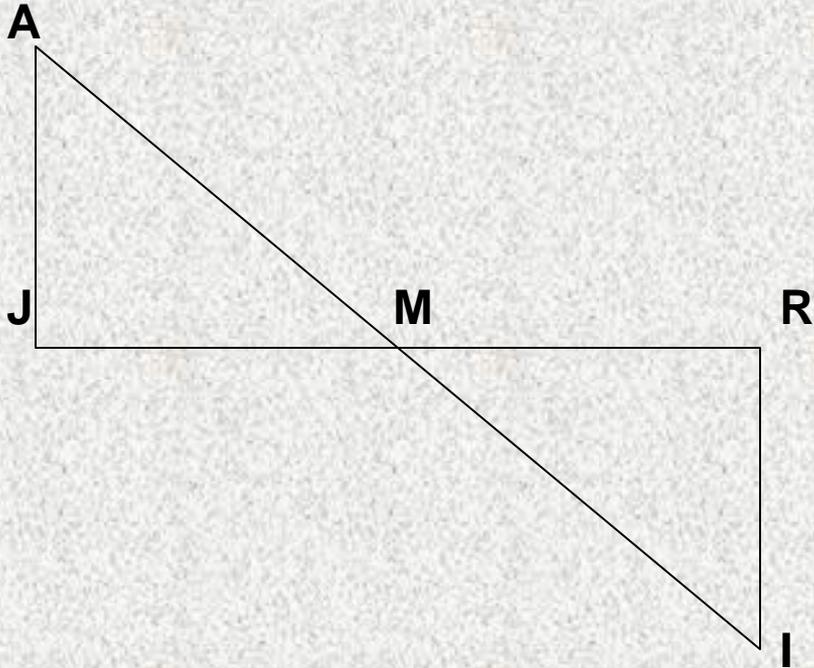
# CPCTC in a Proof:



**Given:**  $TI \cong SN$   
 $TN \cong SI$

**Prove:**  $\angle T \cong \angle S$

# CPCTC in Proofs:



**Given:**  $JM \cong RM$   
 $AM \cong MI$

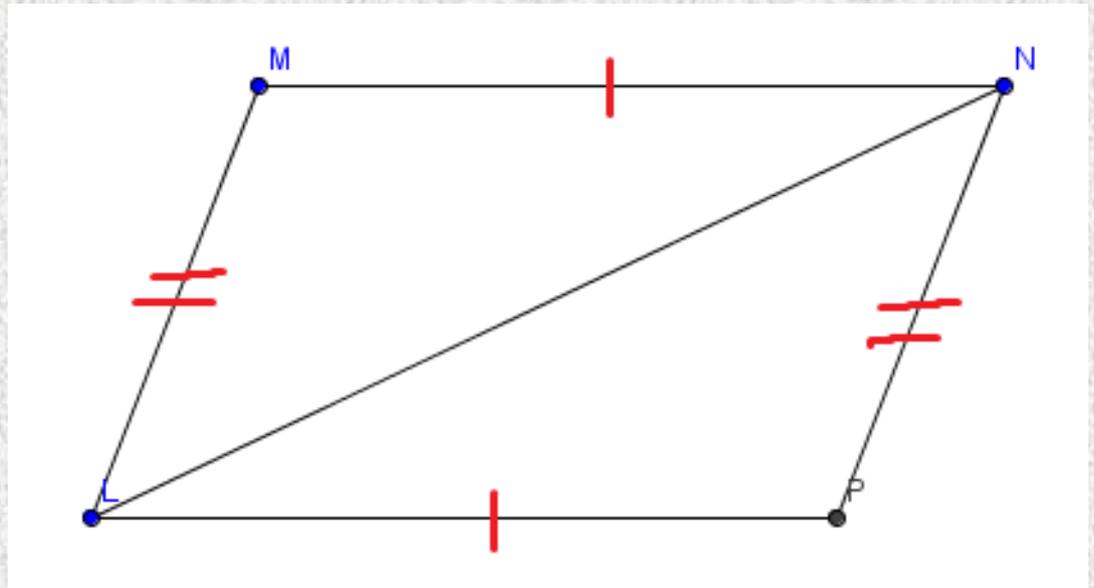
**Prove:**  $AJ \cong RI$

Write a proof.

**Given:**  $\overline{LM} \cong \overline{NP}$ ;

$\overline{MN} \cong \overline{PL}$

**Prove:**  $\triangle LMN \cong \triangle NPL$

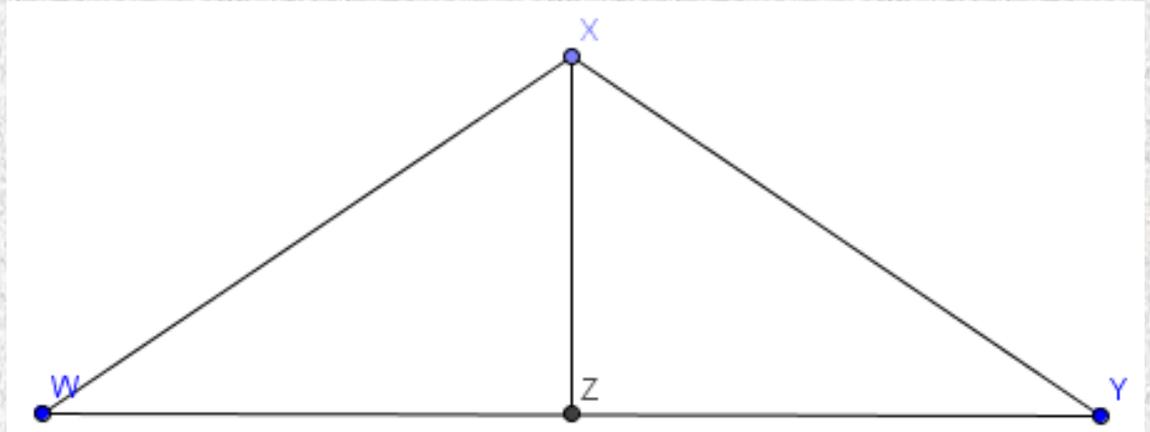


STATEMENTS	REASONS
1. $\overline{LM} \cong \overline{NP}$	1. Given
2. $\overline{MN} \cong \overline{PL}$	2. Given
3. $\overline{LN} \cong \overline{LN}$	3. Reflexive Prop.
4. $\triangle LMN \cong \triangle NPL$	4. SSS Congruence Post.

Write a proof.

**Given:**  $Z$  is the midpoint  
of  $\overline{WY}$ .  
 $\angle XZY = 90^\circ$

**Prove:**  $\triangle WXZ \cong \triangle YXZ$



$Z$  is the midpoint  
of  $\overline{WY}$ .  
**Given**

$\overline{WZ} \cong \overline{ZY}$   
**Def. of  
midpt.**

$\angle XZY = 90^\circ$   
**Given**

$\angle XZW = 90^\circ$   
**Linear pair**

$\overline{XZ} \cong \overline{XZ}$   
**Reflex.  
Prop.**

$\triangle WXZ \cong \triangle YXZ$   
**SAS Cong.  
Post.**

# Assignment

Exercises Pg. 204-207 #1-27 odd

# **Isosceles and Equilateral Triangles**

## **Section 4-5**

## ***Vocabulary***

**legs of an isosceles triangle**

**vertex angle**

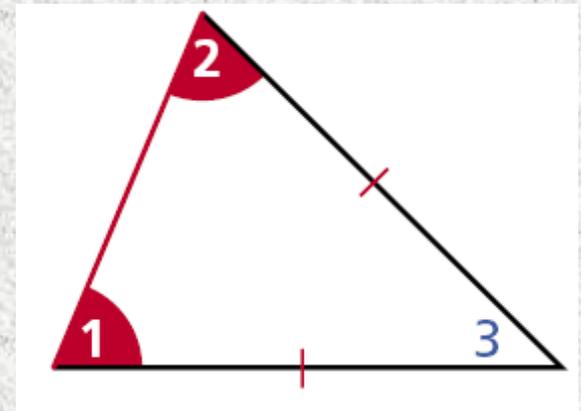
**base**

**base angles**

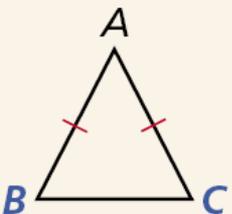
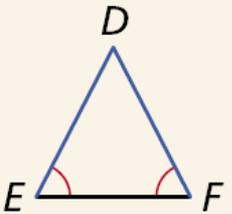
Recall that an isosceles triangle has at least two congruent sides. The congruent sides are called the legs. The vertex angle is the angle formed by the legs. The side opposite the vertex angle is called the base, and the base angles are the two angles that have the base as a side.

$\angle 3$  is the vertex angle.

$\angle 1$  and  $\angle 2$  are the base angles.



**Theorems****Isosceles Triangle**

THEOREM	HYPOTHESIS	CONCLUSION
<b>4-8-1 Isosceles Triangle Theorem</b> If two sides of a triangle are congruent, then the angles opposite the sides are congruent.		$\angle B \cong \angle C$
<b>4-8-2 Converse of Isosceles Triangle Theorem</b> If two angles of a triangle are congruent, then the sides opposite those angles are congruent.		$\overline{DE} \cong \overline{DF}$

## Reading Math

**The Isosceles Triangle Theorem is sometimes stated as “Base angles of an isosceles triangle are congruent.”**

## Example 1: Astronomy Application

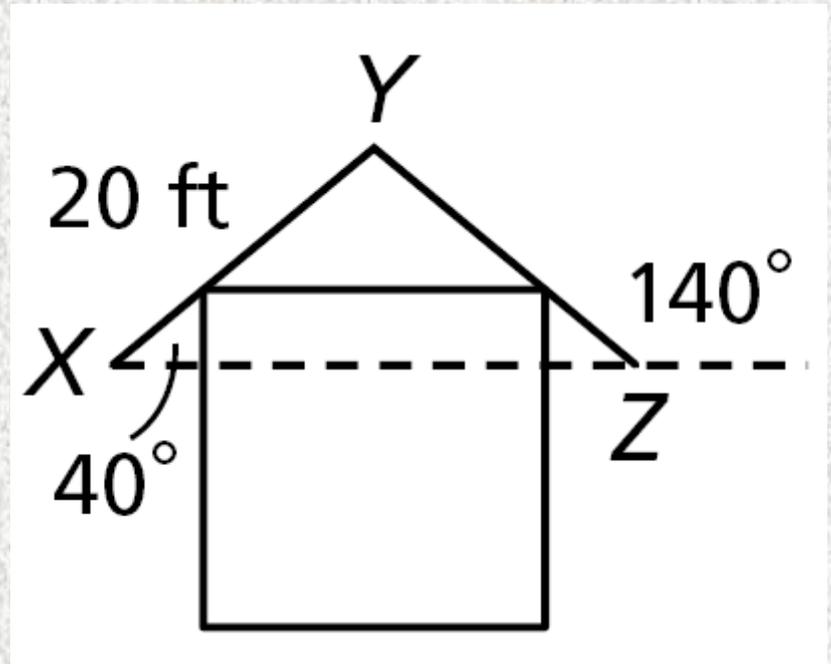
The length of  $YX$  is  $20 \overline{\text{feet}}$ .

Explain why the length of  $YZ$  is the same.——

The  $m\angle YZX = 180 - 140$ , so  $m\angle YZX = 40^\circ$ .

Since  $\angle YZX \cong \angle X$ ,  $\triangle XYZ$  is isosceles by the Converse of the Isosceles Triangle Theorem.

Thus  $YZ = YX = 20 \text{ ft.}$



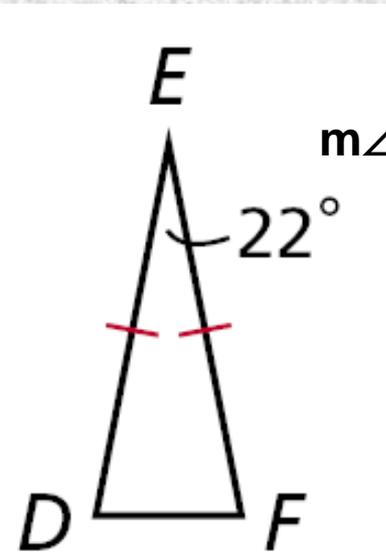
## **Example 1**

**If the distance from Earth to a star in September is  $4.2 \times 10^{13}$  km, what is the distance from Earth to the star in March? Explain.**

**$4.2 \times 10^{13}$ ; since there are 6 months between September and March, the angle measures will be approximately the same between Earth and the star. By the Converse of the Isosceles Triangle Theorem, the triangles created are isosceles, and the distance is the same.**

## Example 2A: Finding the Measure of an Angle

Find  $m\angle F$ .



$$m\angle F = m\angle D = x^\circ$$

*Isosc.  $\triangle$  Thm.*

$$m\angle F + m\angle D + m\angle A = 180$$

*$\triangle$  Sum Thm.*

$$x + x + 22 = 180$$

*Substitute the given values.*

$$2x = 158$$

*Simplify and subtract 22 from both sides.*

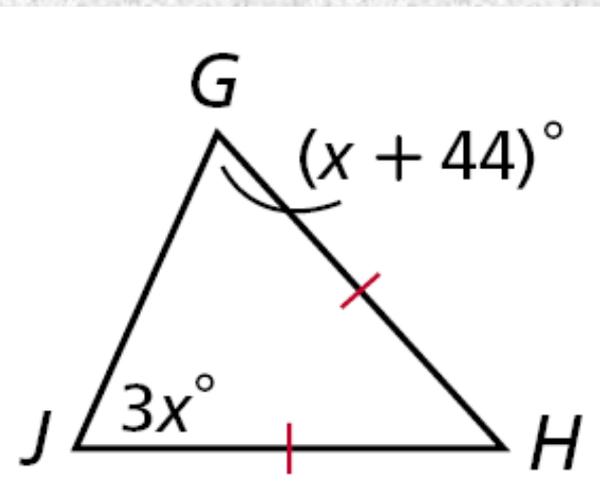
$$x = 79^\circ$$

*Divide both sides by 2.*

Thus  $m\angle F = 79^\circ$

## Example 2B: Finding the Measure of an Angle

Find  $m\angle G$ .



$$m\angle J = m\angle G$$

*Isosc.  $\Delta$  Thm.*

$$(x + 44)^\circ = 3x^\circ$$

*Substitute the given values.*

$$44 = 2x$$

*Simplify  $x$  from both sides.*

$$x = 22^\circ$$

*Divide both sides by 2.*

$$\text{Thus } m\angle G = 22^\circ + 44^\circ = 66^\circ.$$

## Example 2A

Find  $m\angle H$ .

$$m\angle H = m\angle G = x^\circ$$

$$m\angle H + m\angle G + m\angle F = 180$$

$$x + x + 48 = 180$$

$$2x = 132$$

$$x = 66^\circ$$

Thus  $m\angle H = 66^\circ$

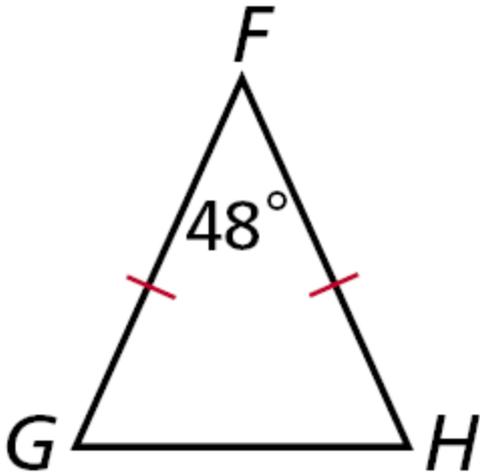
*Isosc.  $\Delta$  Thm.*

*$\Delta$  Sum Thm.*

*Substitute the given values.*

*Simplify and subtract 48 from both sides.*

*Divide both sides by 2.*



## Check It Out! Example 2B

Find  $m\angle N$ .

$$m\angle P = m\angle N$$

*Isosc.  $\Delta$  Thm.*

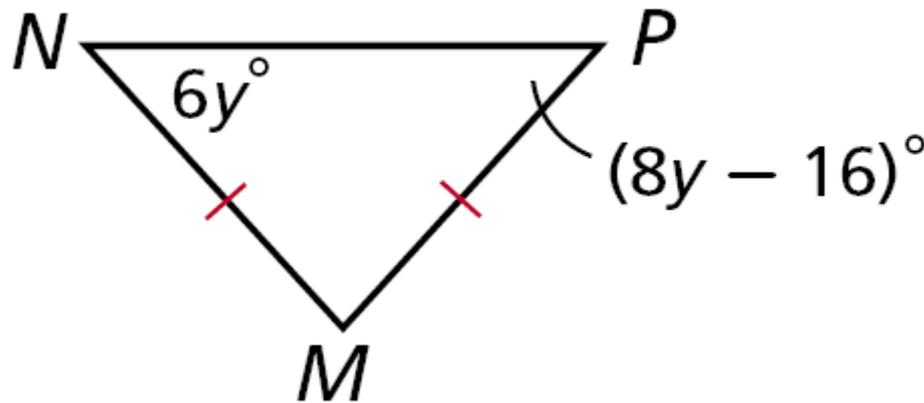
$$(8y - 16)^\circ = 6y^\circ \quad \text{Substitute the given values.}$$

$$2y = 16$$

*Subtract  $6y$  and add 16 to both sides.*

$$y = 8^\circ$$

*Divide both sides by 2.*



$$\text{Thus } m\angle N = 6(8) = 48^\circ.$$

The following corollary and its converse show the connection between equilateral triangles and equiangular triangles.

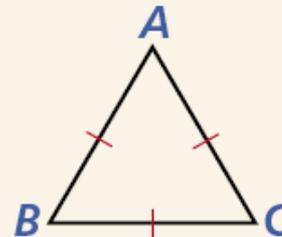
**Corollary 4-8-3** Equilateral Triangle

**COROLLARY**

If a triangle is equilateral, then it is equiangular.  
(equilateral  $\triangle \rightarrow$  equiangular  $\triangle$ )



**HYPOTHESIS**



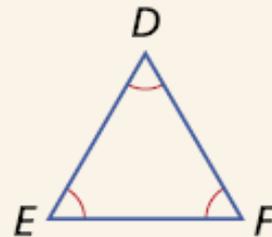
**CONCLUSION**

$$\angle A \cong \angle B \cong \angle C$$

**Corollary 4-8-4****Equiangular Triangle****COROLLARY**

If a triangle is equiangular, then it is equilateral.

(equiangular  $\triangle \rightarrow$  equilateral  $\triangle$ )

**HYPOTHESIS****CONCLUSION**

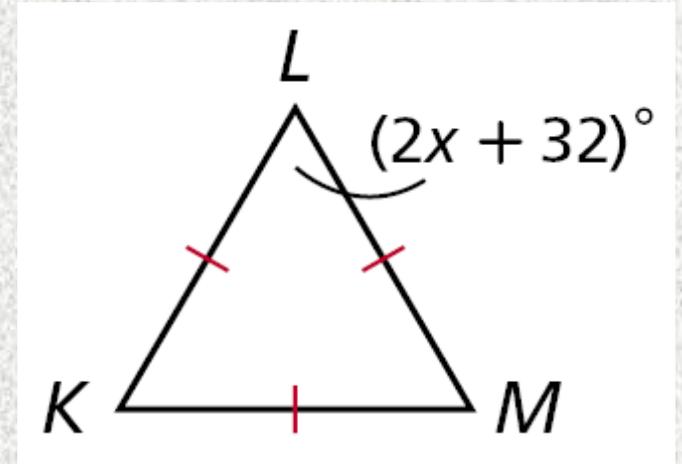
$$\overline{DE} \cong \overline{DF} \cong \overline{EF}$$

## Example 3A: Using Properties of Equilateral Triangles

Find the value of  $x$ .

$\triangle LKM$  is equilateral.

*Equilateral  $\triangle \rightarrow$  equiangular  $\triangle$*



$$(2x + 32)^\circ = 60^\circ$$

*The measure of each  $\angle$  of an equiangular  $\triangle$  is  $60^\circ$ .*

$$2x = 28$$

*Subtract 32 both sides.*

$$x = 14$$

*Divide both sides by 2.*

### Example 3B: Using Properties of Equilateral Triangles

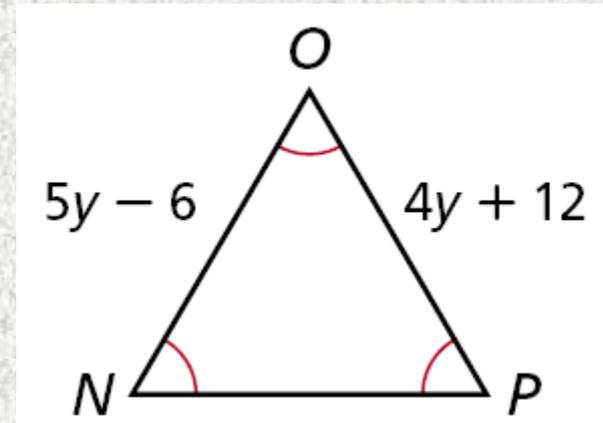
Find the value of  $y$ .

$\triangle NPO$  is equiangular.

*Equiangular  $\triangle \rightarrow$  equilateral  $\triangle$*

$$5y - 6 = 4y + 12$$

$$y = 18$$



*Definition of equilateral  $\triangle$ .*

*Subtract  $4y$  and add 6 to both sides.*

### Check It Out! Example 3

Find the value of JL.

$\triangle JKL$  is equiangular.

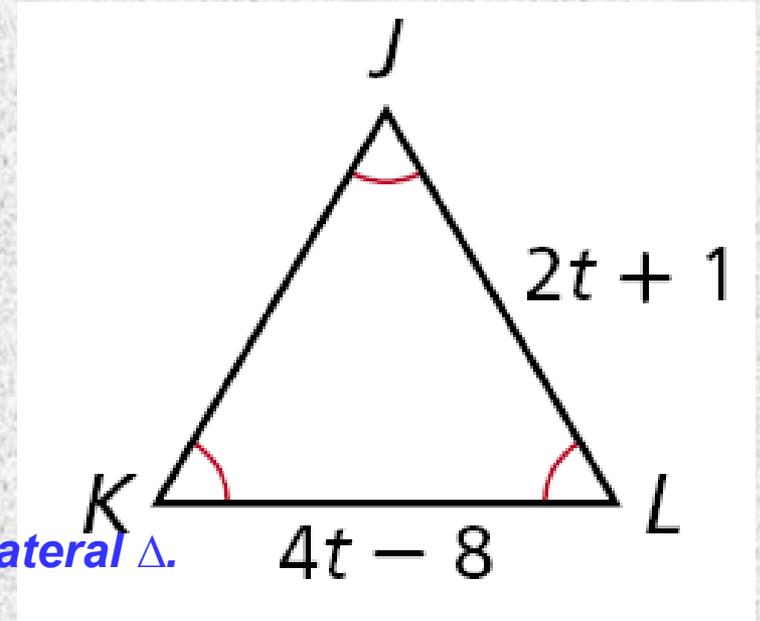
*Equiangular  $\triangle \rightarrow$  equilateral  $\triangle$*

$$4t - 8 = 2t + 1$$

$$2t = 9$$

$$t = 4.5$$

$$\text{Thus } JL = 2(4.5) + 1 = 10.$$



*Definition of equilateral  $\triangle$ .*

*Subtract 4y and add 6 to both sides.*

*Divide both sides by 2.*

## Remember!

**A coordinate proof may be easier if you place one side of the triangle along the  $x$ -axis and locate a vertex at the origin or on the  $y$ -axis.**

## Example 4: Using Coordinate Proof

Prove that the segment joining the midpoints of two sides of an isosceles triangle is half the base.

Given: In isosceles  $\triangle ABC$ ,  $X$  is the mdpt. of  $AB$ , and  $Y$  is the mdpt. of  $AC$ .

Prove:  $XY = AC \cdot \frac{1}{2}$

## Example 4 Continued

**Proof:**

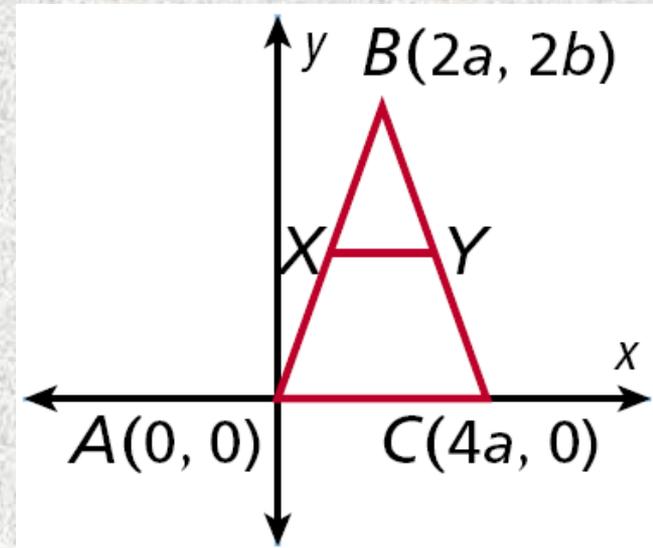
**Draw a diagram and place the coordinates as shown.**

**By the Midpoint Formula, the coordinates of  $X$  are  $(a, b)$ , and  $Y$  are  $(3a, b)$ .**

**By the Distance Formula,  $XY = \sqrt{4a^2} = 2a$ , and  $AC = 4a$ .**

**Therefore  $\overline{XY} = \frac{1}{2} \overline{AC}$ .**

$$\frac{1}{2}$$

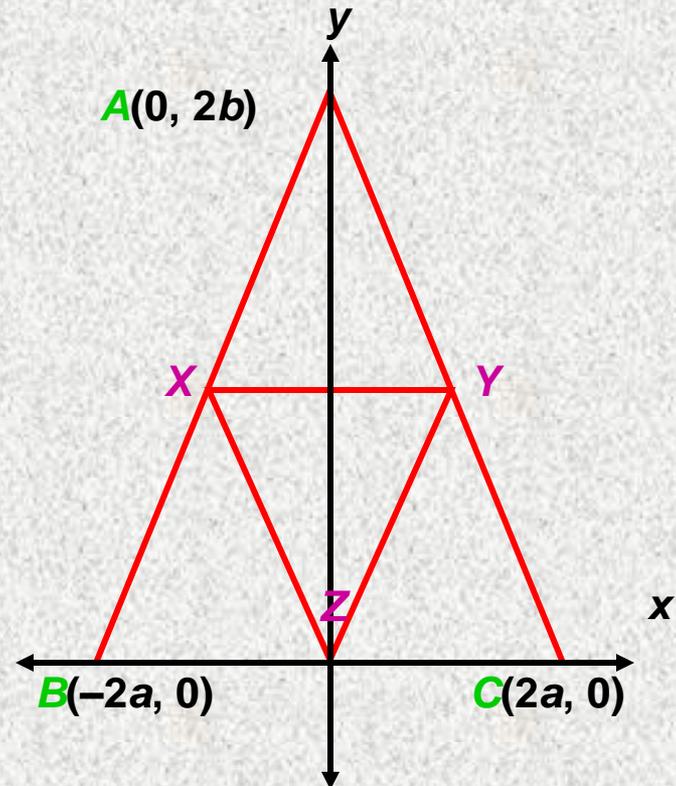


### Example 4

**What if...?** The coordinates of isosceles  $\triangle ABC$  are  $A(0, 2b)$ ,  $B(-2a, 0)$ , and  $C(2a, 0)$ .  $X$  is the midpoint of  $AB$ , and  $Y$  is the midpoint of  $AC$ . Prove  $\triangle XYZ$  is isosceles.

**Proof:**

Draw a diagram and place the coordinates as shown.

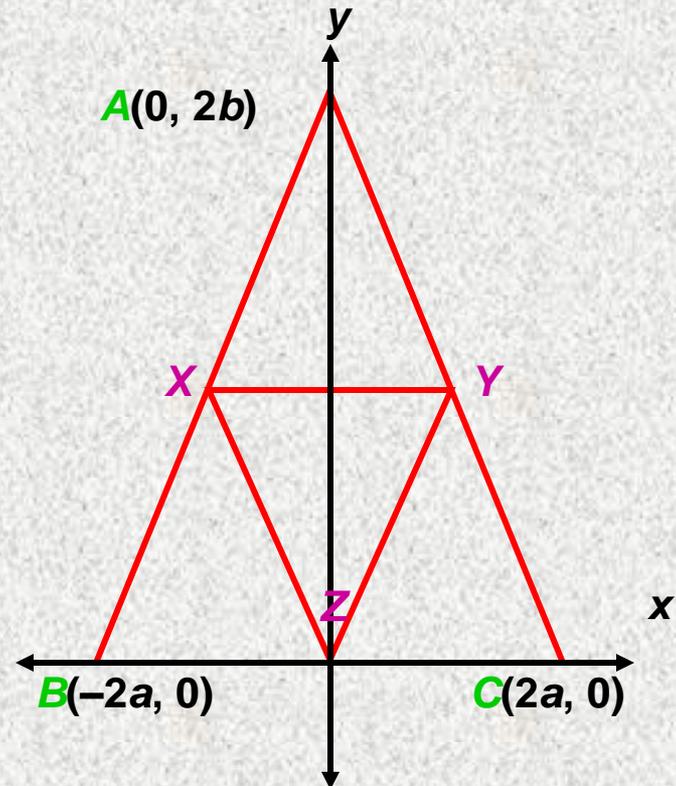


## Example 4 Continued

By the Midpoint Formula, the coordinates of  $X$  are  $(-a, b)$ , the coordinates of  $Y$  are  $(a, b)$ , and the coordinates of  $Z$  are  $(0, 0)$ . By the Distance Formula,  $XZ = YZ = \sqrt{a^2 + b^2}$ .

So  $XZ \cong YZ$  and  $\triangle XYZ$  is isosceles. \_\_\_\_\_

\_\_\_\_\_

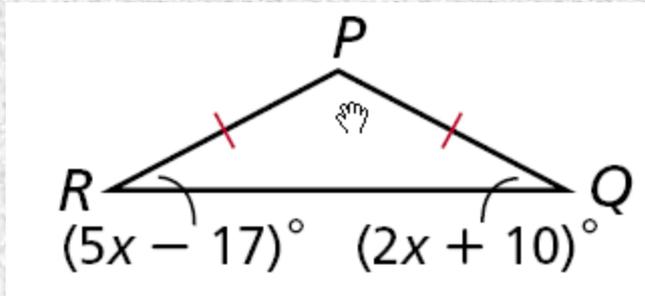


## Lesson Quiz: Part I

Find each angle measure.

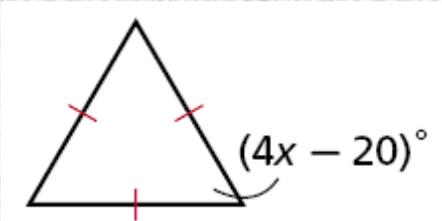
1.  $m\angle R$                        **$28^\circ$**

2.  $m\angle P$                        **$124^\circ$**



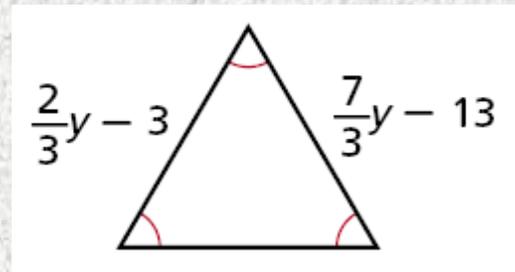
Find each value.

3.  $x$



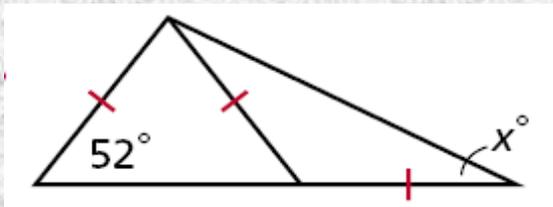
**20**

4.  $y$



**6**

5.  $x$



**$26^\circ$**

## Lesson Quiz: Part II

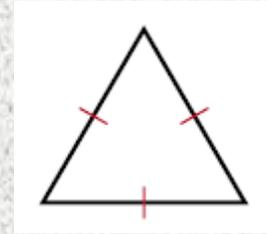
6. The vertex angle of an isosceles triangle measures  $(a + 15)^\circ$ , and one of the base angles measures  $7a^\circ$ . Find  $a$  and each angle measure.

$$a = 11; 26^\circ; 77^\circ; 77^\circ$$

## Warm Up

1. Find each angle measure.

$60^\circ$ ;  $60^\circ$ ;  $60^\circ$



True or False. If false explain.

2. Every equilateral triangle is isosceles.

True

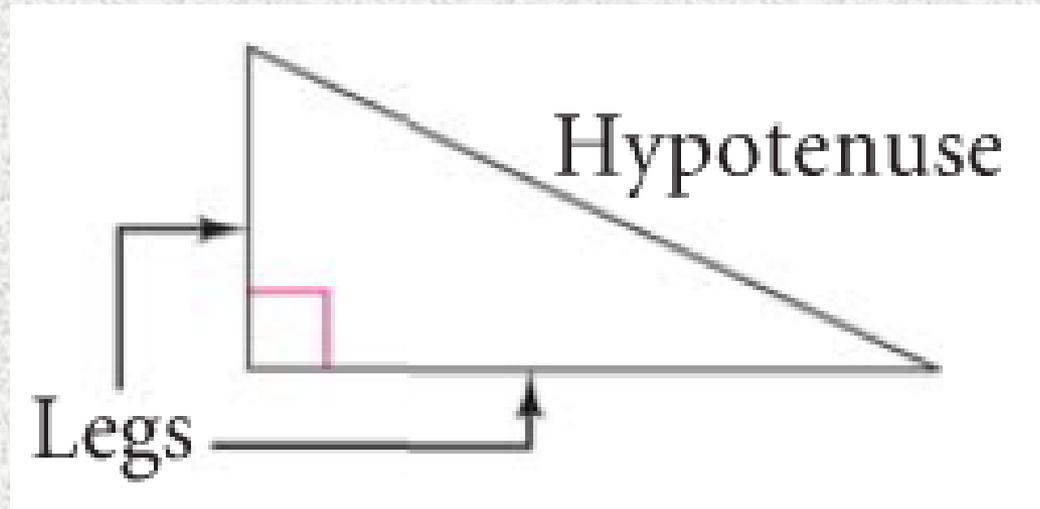
3. Every isosceles triangle is equilateral.

False; an isosceles triangle can have only two congruent sides.

# **Congruence in Right Triangles**

Section 4-6

# Right Triangles



**Hypotenuse:** the longest side of a right triangle

**Legs:** The sides of a right triangle that are not the hypotenuse

# Theorem 😊

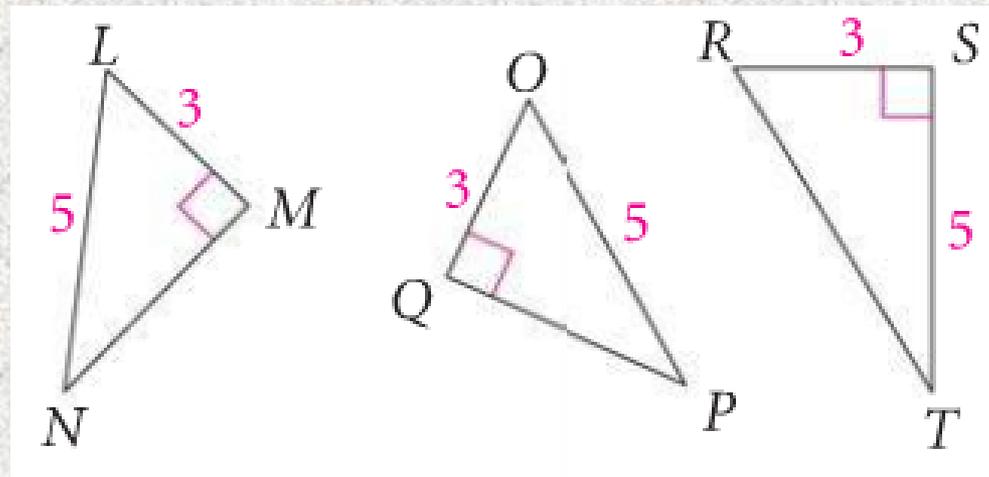
**Theorem 4.6: Hypotenuse-Leg (HL) Theorem: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.**

**To use the HL Theorem, you must show that three conditions are met:**

- **There are two right triangles**
- **The triangles have congruent hypotenuses**
- **There is one pair of congruent legs**

# Using the HL Theorem

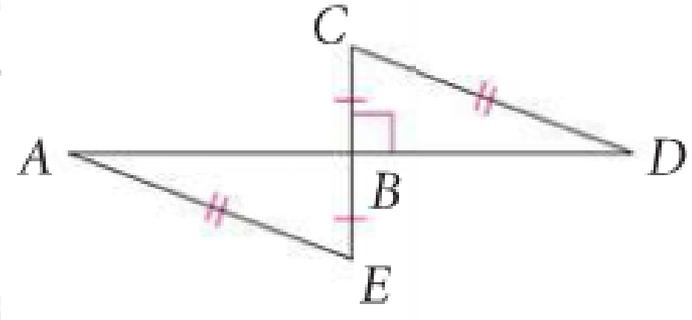
Which two triangles are congruent by the HL Theorem? Write a correct congruence statement.



# Using the HL Theorem

**Given:**  $\overline{CD} \cong \overline{EA}$ ,  $\overline{AD}$  is the perpendicular bisector of  $\overline{CE}$ .

**Prove:**  $\triangle CBD \cong \triangle EBA$

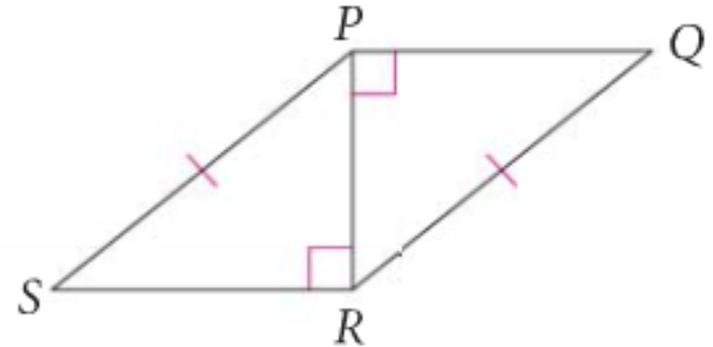


Statements	Reasons
1. $\overline{AD}$ is the $\perp$ bisector of $\overline{CE}$ , $\overline{CD} \cong \overline{EA}$	1.
2.	2. Defn. of $\perp$
3. $\triangle CBD$ & $\triangle EBA$ are rt. $\Delta$ s	3.
4. $\overline{CB} \cong \overline{EB}$	4.
5.	5. HL Thm.

# Using the HL Theorem

**Given:**  $\angle PRS$  and  $\angle RPQ$  are right angles,  
 $\overline{SP} \cong \overline{QR}$ .

**Prove:**  $\triangle PRS \cong \triangle RPQ$



Statements	Reasons
1. $\angle PRS$ and $\angle RPQ$ are rt $\angle$ s $\overline{SP} \cong \overline{QR}$	1.
2.	2. Defn. of rt $\Delta$ s
3.	3. Refl. Prop. of $\cong$
4. $\triangle PRS \cong \triangle RPQ$	4.

**Practice!**

**Pg. 219-221 #1-23 odd**

