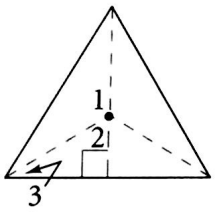
A Practice by Example

Example 1 (page 380)

Each regular polygon has radii and apothem as shown. Find the measure of each numbered angle.

1.



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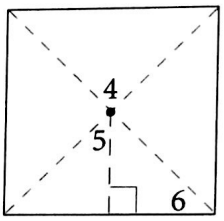

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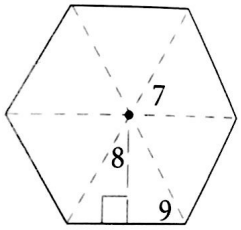


Example 2 (page 381)

Find the area of each regular polygon with the given apothem a and side length s.

4. pentagon, $a = 24.3$ cm, $s = 35.3$ cm

5. 7-gon, $a = 29.1$ ft, $s = 28$ ft

6. octagon, $a = 60.4$ in., $s = 50$ in.

7. nonagon, $a = 27.5$ in., $s = 20$ in.

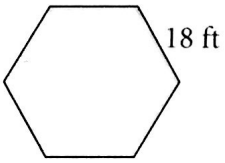
8. decagon, $a = 19$ m, $s = 12.3$ m

9. dodecagon, $a = 26.1$ cm, $s = 14$ cm

Example 3 (page 381)

Find the area of each regular polygon. Round your answer to the nearest tenth.

10.



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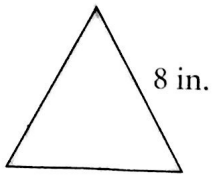

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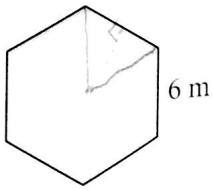
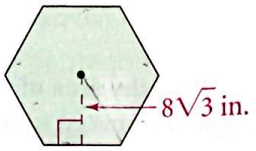


Example 4 (page 382)

Find the area of each regular polygon with the given radius or apothem. If your answer is not an integer, leave it in simplest radical form.

13.

14.



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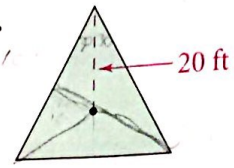

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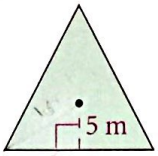

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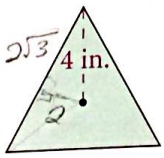
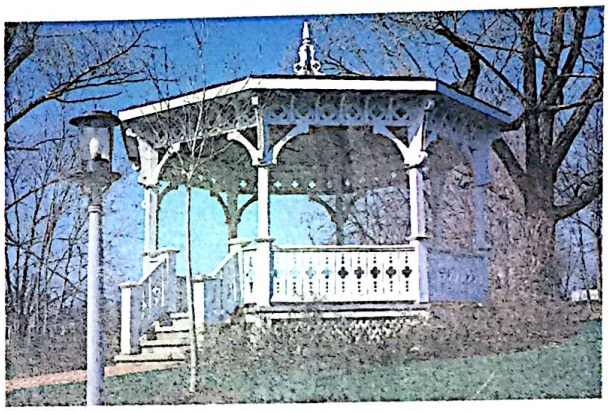


B Apply Your Skills

Find the measures of the angles formed by (a) two consecutive radii and (b) a radius and a side of the given regular polygon.

19. pentagon **20.** octagon **21.** nonagon **22.** dodecagon

23. Architecture The gazebo in the photo is built in the shape of a regular octagon. Each side is 8 ft long, and its apothem is 9.7 ft. To the nearest tenth, find the area enclosed by the gazebo.



24. The area of a regular polygon is 36 in.^{2}. Find the length of a side if the polygon has the given number of sides. Round your answer to the nearest tenth.

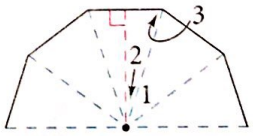
a. 3 **b.** 4 **c.** 6

d. Estimation Suppose the polygon is a pentagon. What would you expect the length of its side to be? Explain.

Need Help?

In Exercise 25, $m\angle 1$ is what part of 360?

25. A portion of a regular decagon has radii and an apothem drawn. Find the measure of each numbered angle.



26. Writing Explain why the radius of a regular polygon is greater than the apothem.

27. Satellites One of the smallest space satellites ever developed has the shape of a pyramid. Each of the four faces of the pyramid is an equilateral triangle with sides about 13 cm long. What is the area of one equilateral triangular face of the satellite? Round your answer to the nearest whole number.

Find the area of each equilateral triangle with the given radius. Round your answers to the nearest whole number.

28. $r = 10$ in. **29.** $r = 4.6$ m **30.** $r = 8.9$ ft **31.** $r = 13$ cm

32. Constructions Use a compass to construct a circle.

a. Construct four perpendicular radii of the circle.

b. Construct radii that bisect each of the four right angles.

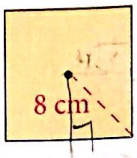
c. Connect the consecutive points where the radii intersect the circle. What regular polygon have you constructed?

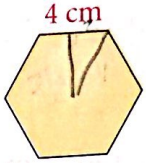
d. Critical Thinking How can a circle help you construct a regular hexagon?

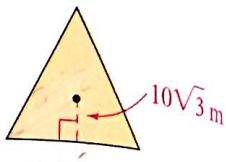
33. A regular hexagon has perimeter 120 m. Find its area.

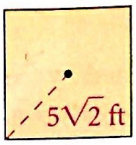
34. **Open-Ended** Create a design using equilateral triangles and regular hexagons that have sides of the same length. Find the area of the completed design.

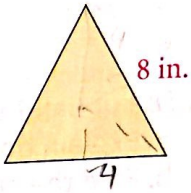
Find the area of each regular polygon. Show your answers in simplest radical form and rounded to the nearest tenth.

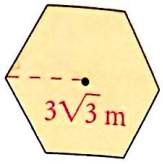
35. 

36. 

37. 

38. 

39. 

40. 

41. To find the area of an equilateral triangle, you can use the formula $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ap$. A third way to find the area of an equilateral triangle is to use the formula $A = \frac{1}{4}s^2\sqrt{3}$. Verify the formula $A = \frac{1}{4}s^2\sqrt{3}$ in two ways as follows:

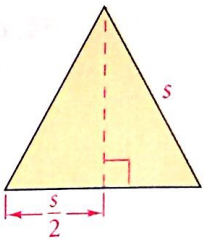


Figure 1

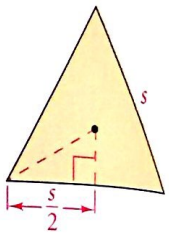


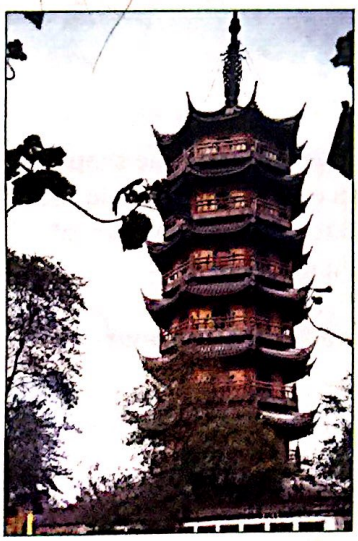
Figure 2

a. Find the area of Figure 1 using the formula $A = \frac{1}{2}bh$.

b. Find the area of Figure 2 using the formula $A = \frac{1}{2}ap$.

Proof 42. For Example 1 on page 380, write a proof that the apothem bisects the vertex angle of the isosceles triangle formed by the radii.

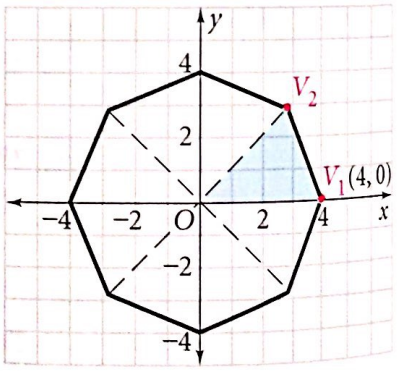
Challenge Proof 43. Prove that the bisectors of the angles of a regular polygon (given congruent sides and angles) are concurrent and that they are, in fact, radii of the polygon. (*Hint:* For regular n-gon $ABCDE\ldots$, let P be the intersection of the bisectors of $\angle ABC$ and $\angle BCD$. Show that $\overrightarrow{DP}$ must be the bisector of $\angle CDE$.)



Real-World Connection

Horizontal cross sections of the Wenfeng Pagoda in Yangzhou, China, are regular octagons.

44. **Coordinate Geometry** A regular octagon with center at the origin and radius 4 is graphed in the coordinate plane.



a. Since V_2 lies on the line $y = x$, its x- and y-coordinates are equal. Use the Distance Formula to find the coordinates of V_2 to the nearest tenth.

b. Use the coordinates of V_2 and the formula $A = \frac{1}{2}bh$ to find the area of $\triangle V_1OV_2$ to the nearest tenth.

c. Use your answer to part (b) to find the area of the octagon to the nearest whole number.

45. **a.** Find the area of the triangle if the area of each square is 10 cm^2.

b. When a square and an equilateral triangle share a common side, what is the ratio of the area of the triangle to the area of the square? Leave your answer in simplest radical form.