Practice and Problem Solving

A Practice by Example

Example 1 (page 264)

Write the negation of each statement.

1. Two angles are congruent.
2. You are not sixteen years old.
3. The angle is not obtuse.
4. The soccer game is on Friday.
5. The figure is a triangle.
6. $m\angle A < 90$

Example 2 (page 265)

Write (a) the inverse and (b) the contrapositive of each conditional statement.

7. If you eat all of your vegetables, then you will grow.
8. If a figure is a square, then all of its angles are right angles.
9. If a figure is a rectangle, then it has four sides.

Example 3 (page 266)

Developing Proof **Write the first step of an indirect proof.**

10. It is raining outside.
11. $\angle J$ is not a right angle.
12. $\triangle PEN$ is isosceles.
13. At least one angle is obtuse.
14. $\overline{XY} \cong \overline{AB}$
15. $m\angle 2 > 90$

Example 4 (page 266)

Developing Proof **Identify the two statements that contradict each other.**

16. I. $\triangle PQR$ is equilateral.
    II. $\triangle PQR$ is a right triangle.
    III. $\triangle PQR$ is isosceles.
17. I. In right $\triangle ABC, m\angle A = 60$.
    II. In right $\triangle ABC, \angle A \cong \angle C$.
    III. In right $\triangle ABC, m\angle B = 90$.
18. I. ℓ and m are skew.
    II. ℓ and m do not intersect.
    III. $\ell \parallel m$
19. I. Each of the two items that Val bought costs more than \$10.
    II. Val spent \$34 for the two items.
    III. Neither of the two items that Val bought costs more than \$15.

Example 5 (page 266)

20. Developing Proof Fill in the blanks to prove the following statement.
If the Debate and Chess Clubs together have fewer than 20 members and the Chess Club has 10 members, then the Debate Club has fewer than 10 members.

Given: The total membership of the Debate Club and the Chess Club is fewer than 20. The Chess Club has 10 members.

Prove: The Debate Club has fewer than 10 members.

Proof: Assume that the Debate Club has 10 or more members.
This means that together the two clubs have **a.** ? members.
This contradicts the given information that **b.** ?.
The assumption is false. Therefore it is true that **c.** ?.

Reading Math
For help with reading and solving Exercise 21, see p. 271.

21. Developing Proof Fill in the blanks to prove the following statement.
In a given triangle, $\triangle LMN$, there is at most one right angle.

Given: $\triangle LMN$

Prove: $\triangle LMN$ has at most one right angle.

Proof: Assume that $\triangle LMN$ has more than one **a.** ?. That is, assume that both $\angle M$ and $\angle N$ are **b.** ?. If $\angle M$ and $\angle N$ are both right angles, then $m\angle M = m\angle N =$ **c.** ?. By the Triangle Angle-Sum Theorem, $m\angle L + m\angle M + m\angle N =$ **d.** ?. Use substitution to find $m\angle L +$ **e.** ? $+$ **f.** ? $= 180$. When you solve for $m\angle L$, you find that $m\angle L =$ **g.** ?. This means that there is no $\triangle LMN$, which contradicts the given statement. So the assumption that $\triangle LMN$ has **h.** ? must be false. Therefore, $\triangle LMN$ has **i.** ?.

B Apply Your Skills

Write (a) the inverse and (b) the contrapositive of each statement. Give the truth value of each.

22. If you live in Sarasota, then you live in Florida.

23. If four points are collinear, then they are coplanar.

Open-Ended **Write a true conditional statement for each given condition. If such a statement is not possible, tell why.**

24. The inverse is false.

25. The inverse is true.

26. The contrapositive is false.

27. The contrapositive is true.

Real-World Connection
Water freezes at 32°F. Sidewalk "salt" lowers the freezing point of water.

28. Error Analysis Angie saw an ad that stated "If you don't drink Muscle Rex, then you won't build muscles." Angie bought Muscle Rex and drank it, and nothing happened. She sent an e-mail to the company asking for her money back. The company would not refund her money. They claimed that her reasoning was faulty. Using one or more of the terms *converse*, *inverse*, or *contrapositive*, explain why Angie's reasoning was faulty.

Writing **For Exercises 29–32, write a convincing argument that uses indirect reasoning.**

29. Fresh skid marks appear behind a green car at the scene of an accident. Show that the driver of the green car applied the brakes.

30. Ice is forming on the sidewalk in front of Toni's house. Show that the temperature of the sidewalk surface must be 32°F or lower.

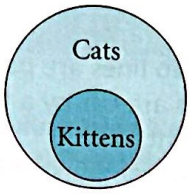
31. An obtuse triangle cannot contain a right angle.

32. In a plane, a line has no more than one perpendicular at any of its points.

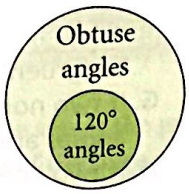
Need Help?
To review Venn diagrams, see page 69.

Write the conditional statement illustrated by each Venn diagram. Then write its contrapositive.

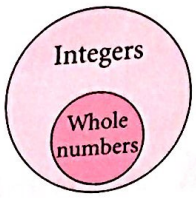
33.

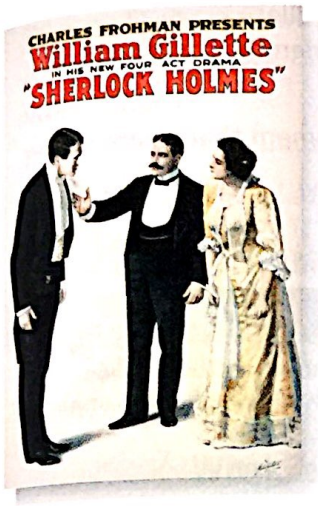


34.



35.





Real-World Connection

The key to Sherlock Holmes's success is his use of deductive reasoning.

36. Open-Ended Describe a real-life situation in which you used an indirect argument to convince someone of your point of view. Outline your argument.

37. Earl lives near a noisy construction site at which work ends promptly at 5:00 each workday. Earl thinks, "Today is Tuesday. If it were before 5:00, I would hear construction noise, but I don't hear any. So it must be later than 5:00."

a. What does Earl prove? **b.** What assumption does he make?

c. What fact would contradict the assumption?

38. Literature In Arthur Conan Doyle's story "The Sign of the Four," Sherlock Holmes talks to his friend Watson about how a culprit enters a room that has only four entrances: a door, a window, a chimney, and a hole in the roof.

> "You will not apply my precept," he said, shaking his head. "How often have I said to you that when you have eliminated the impossible, whatever remains, however improbable, must be the truth? We know that he did not come through the door, the window, or the chimney. We also know that he could not have been concealed in the room, as there is no concealment possible. Whence, then, did he come?"

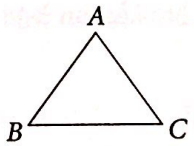
How did the culprit enter the room? Explain.

Challenge

Proof 39. Use indirect reasoning to prove the following.

Given: $\triangle ABC$ with $BC > AC$

Prove: $\angle A \ncong \angle B$



Proof 40. Write an indirect proof.

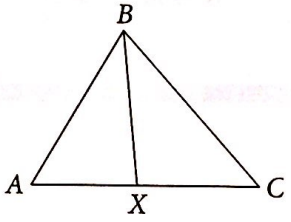
Given: $\triangle XYZ$ is isosceles.

Prove: Neither base angle is a right angle.

Proof 41. Write an indirect proof.

Given: $\triangle ABC$ is scalene, $m\angle ABX = 36$, and $m\angle CBX = 36$.

Prove: $\overline{XB}$ is not perpendicular to $\overline{AC}$.



Standardized Test Prep