EXERCISES

Practice and Problem Solving

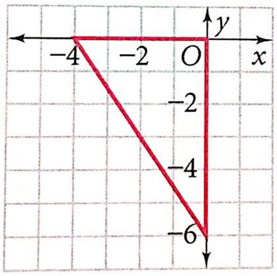
For more practice, see *Extra Practice*.

A Practice by Example

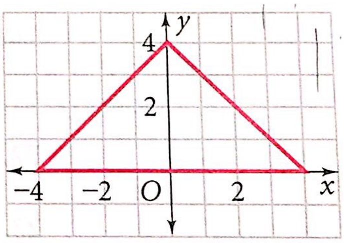
Example 1
(page 257)

Coordinate Geometry **Find the center of the circle that you can circumscribe about each triangle.**

1.



2.



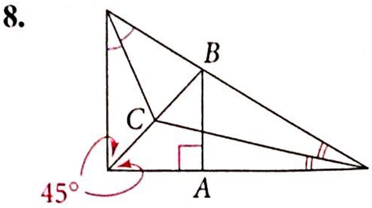
Coordinate Geometry **Find the center of the circle that you can circumscribe about $\triangle ABC$.**

3. $A(0, 0)$	**4.** $A(0, 0)$	**5.** $A(-4, 5)$	**6.** $A(-1, -2)$	**7.** $A(1, 4)$
$B(3, 0)$	$B(4, 0)$	$B(-2, 5)$	$B(-5, -2)$	$B(1, 2)$
$C(3, 2)$	$C(4, -3)$	$C(-2, -2)$	$C(-1, -7)$	$C(6, 2)$

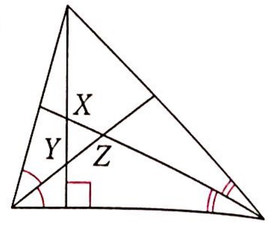
Example 2 (page 258)

Name the point of concurrency of the angle bisectors.

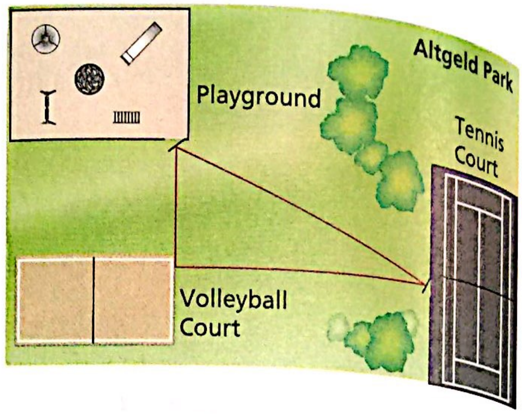
8.



9.

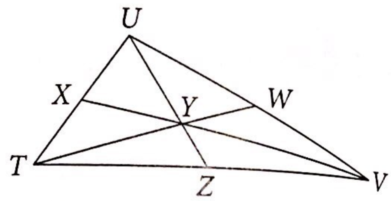


10. City Planning Copy the diagram of Altgeld Park. Show where park officials should place a drinking fountain so that it is equidistant from the tennis court, the playground, and the volleyball court.



Example 3 (page 258)

In $\triangle TUV$, Y is the centroid.



11. If $YW = 9$, find TY and TW.

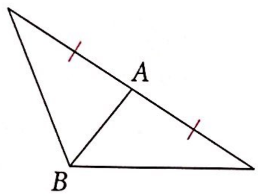
12. If $YU = 9$, find ZY and ZU.

13. If $VX = 9$, find VY and YX.

Example 4 (page 259)

Is $\overline{AB}$ a median, an altitude, or neither? Explain.

14.



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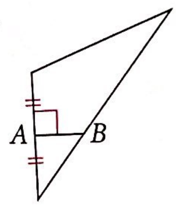

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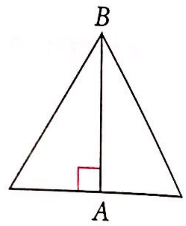


B Apply Your Skills

Constructions **Draw the triangle. Then construct the inscribed circle and the circumscribed circle.**

17. right triangle, $\triangle DEF$

18. obtuse triangle, $\triangle STU$

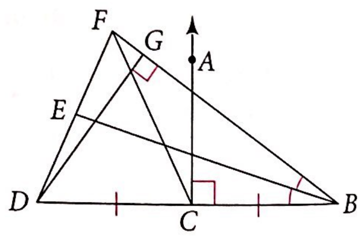
In Exercises 19–22, name each figure in $\triangle BDF$.

19. an angle bisector

20. a median

21. a perpendicular bisector

22. an altitude



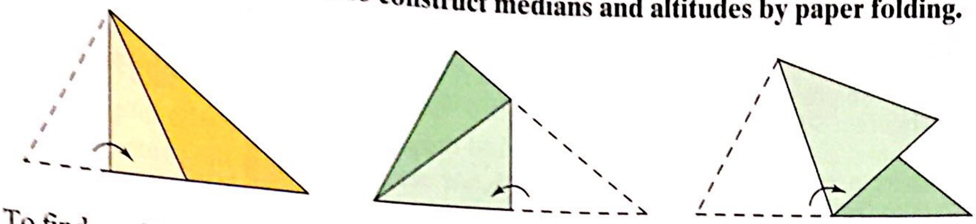
23. Critical Thinking A centroid separates a median into two segments. What is the ratio of the lengths of those segments?

24. Writing Ivars found a yellowed parchment inside an antique book. It read:

From the spot I buried Olaf's treasure, equal sets of paces did I measure; each of three directions in a line, there to plant a seedling Norway pine. I could not return for failing health; now the hounds of Haiti guard my wealth.—Karl

After searching Caribbean islands for five years, Ivars found one with three tall Norway pines. How might Ivars find where Karl buried Olaf's treasure?

The figures below show how to construct medians and altitudes by paper folding.



To find an altitude, fold the triangle so that a side overlaps itself and the fold contains the opposite vertex.

To find a median, fold one vertex to another vertex. This locates the midpoint of a side.

Then fold so that the fold contains the midpoint and the opposite vertex.

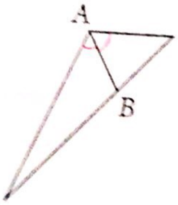
Need Help?
Paper-folding an altitude is the same as paper-folding the perpendicular to a line through a point not on the line.

25. Cut out a large triangle. Paper-fold very carefully to construct the three medians of the triangle and demonstrate Theorem 5-8.

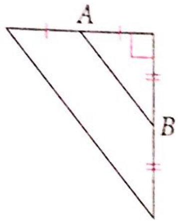
26. Cut out a large acute triangle. Paper-fold very carefully to construct the three altitudes of the triangle and demonstrate Theorem 5-9.

Is $\overline{AB}$ a perpendicular bisector, an angle bisector, a median, an altitude, or none of these? Explain.

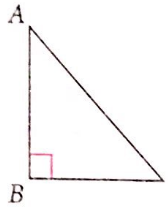
27.



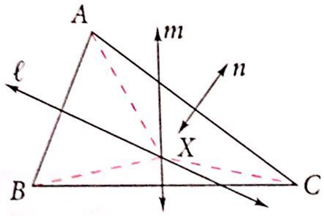
28.



29.



Proof **30. Developing Proof** Complete this proof of Theorem 5-6 by filling in the blanks.

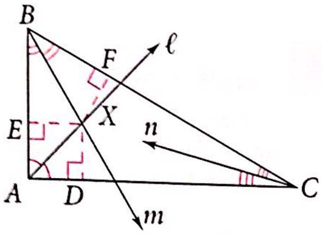


Given: Lines ℓ, m, and n are perpendicular bisectors of the sides of $\triangle ABC$. X is the intersection of lines ℓ and m.

Prove: Line n contains point X, and $XA = XB = XC$.

Proof: Since ℓ is the perpendicular bisector of **a.** _?_, $XA = XB$. Since m is the perpendicular bisector of **b.** _?_, $XB =$ **c.** _?_. Thus $XA = XB = XC$. Since $XA = XC$, X is on line n by the Converse of the **d.** _?_ Theorem.

31. Developing Proof Complete the flow proof of Theorem 5-7.



Given: Rays ℓ, m, and n are bisectors of the angles of $\triangle ABC$. X is the intersection of rays ℓ and m and $\overline{XD} \perp \overline{AC}$, $\overline{XE} \perp \overline{AB}$, $\overline{XF} \perp \overline{BC}$.

Prove: Ray n contains point X, and $XD = XE = XF$.

X is on ray ℓ. (Given) → $XD = XE$ (**a.** _?_) → $XD = XE = XF$ (**c.** _?_ Property of Equality) → X is on ray n. (**d.** Converse of the _?_ Theorem)

X is on ray m. (Given) → $XE = XF$ (**b.** _?_) → $XD = XE = XF$

Reading Math

You can prove Theorem 5-8 for a general $\triangle ABC$ with coordinates $A(0, 0)$, $B(2b, 2c)$, and $C(2c, 0)$ by following the steps for the particular $\triangle ABC$ in Exercise 32.

32. **Coordinate Geometry** Complete the following steps to locate the centroid.

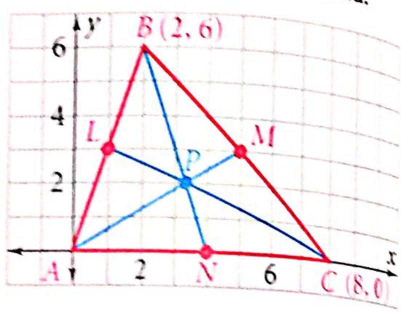
a. Find the coordinates of midpoints L, M, and N.

b. Find equations of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$.

c. Find the coordinates of P, the intersection of $\overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$. This is the centroid.

d. Show that point P is on $\overleftrightarrow{CL}$.

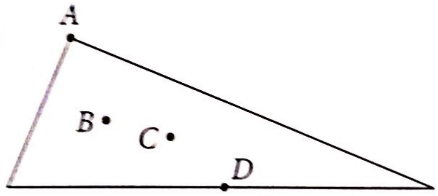
e. Use the Distance Formula to show that point P is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.



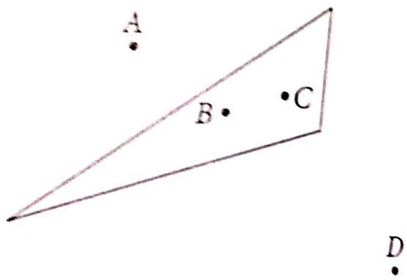
C Challenge

For Exercises 33 and 34, points of concurrency have been drawn for two triangles. Match the points with the lines and segments listed in I–IV.

33.



34.



I. perpendicular bisectors of sides

II. angle bisectors

III. medians

IV. lines containing altitudes

35. In an isosceles triangle, show that the circumcenter, incenter, centroid, and orthocenter can be four different points but all four must be collinear.

36. **History** In 1765 Leonhard Euler proved that for any triangle, three of the four points of concurrency are collinear. The line that contains these three points is known as Euler's Line. Use Exercises 33 and 34 to determine which point of concurrency does not necessarily lie on Euler's Line.

Standardized Test Prep