

Example 1

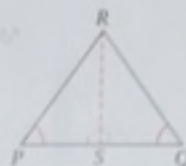
(211)

**1. Developing Proof** Supply the missing parts in this proof of the Converse of the Isosceles Triangle Theorem.

Begin with  $\triangle PRQ$  with  $\angle P = \angle Q$ .  
Draw a.  $\overline{RS}$ , the bisector of  $\angle PRQ$ .

Given:  $\angle P = \angle Q$ , b.  $\overline{RS}$  bisects  $\angle PRQ$ .

Prove:  $\overline{PR} = \overline{QR}$



Statements	Reasons
1. $\overline{RS}$ bisects $\angle PRQ$ .	c. ?
2. $\angle PRS = \angle QRS$	d. ?
3. $\angle P = \angle Q$	3. Given
4. $\overline{RS} = \overline{RS}$	e. ?
5. $\triangle PRS = \triangle QRS$	f. ?
6. $\overline{PR} = \overline{QR}$	6. CPCTC

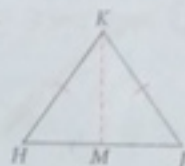
**2. Developing Proof** Here is another way to prove the Isosceles Triangle Theorem. Supply the missing parts.

Begin with isosceles  $\triangle HKJ$  with  $\overline{KH} = \overline{KJ}$ .

Draw a.  $\overline{KM}$ , a bisector of the base  $\overline{HJ}$ .

Given:  $\overline{KH} = \overline{KJ}$ , b.  $\overline{KM}$  bisects  $\overline{HJ}$ .

Prove:  $\angle H = \angle J$



Statements	Reasons
1. $\overline{KM}$ bisects $\overline{HJ}$ .	c. ?
2. $\overline{HM} = \overline{JM}$	d. ?
3. $\overline{KH} = \overline{KJ}$	3. Given
4. $\overline{KM} = \overline{KM}$	e. ?
5. $\triangle KHM = \triangle KJM$	f. ?
6. $\angle H = \angle J$	g. ?

Example 2

(212)

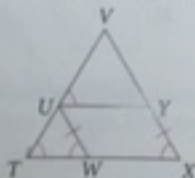
**Developing Proof** Complete each statement. Explain why it is true.

3.  $\overline{VT} = \underline{\hspace{1cm}}$

4.  $\overline{UT} = \underline{\hspace{1cm}} = \overline{YX}$

5.  $\overline{VU} = \underline{\hspace{1cm}}$

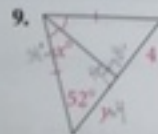
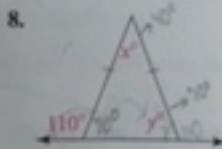
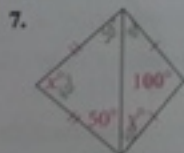
6.  $\angle VYU = \underline{\hspace{1cm}}$



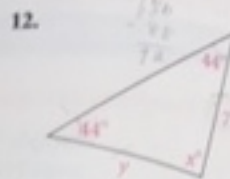
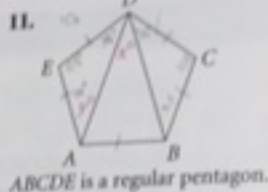
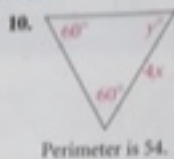
Example 3

(212)

**Algebra** Find the values of  $x$  and  $y$ .



**Algebra** Find the values of  $x$  and  $y$ .



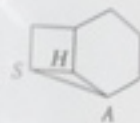
Find each value.

13. If  $m\angle L = 58$ , then  $m\angle LKJ = \square$ .  
 14. If  $JL = 5$ , then  $ML = \square$ .  
 15. If  $m\angle JKM = 48$ , then  $m\angle J = \square$ .  
 16. If  $m\angle J = 55$ , then  $m\angle JKM = \square$ .



Example 4  
p. 212)

17. A square and a regular hexagon are placed so that they have a common side. Find  $m\angle SHA$  and  $m\angle HAS$ .



18. Five fences meet at a point to form angles with measures  $x$ ,  $2x$ ,  $3x$ ,  $4x$ , and  $5x$  around the point. Find the measure of each angle.

Skills

19. **Graphic Arts** The former logo for the National Council of Teachers of Mathematics is shown at the right. Trace the logo onto paper.

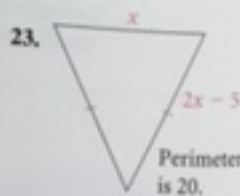
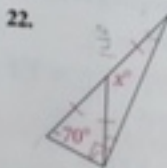
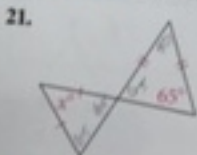


The triangles in the logo have these congruent sides and angles.

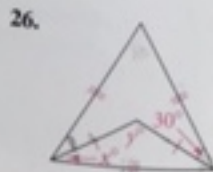
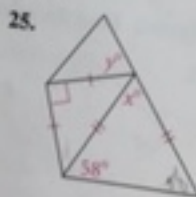
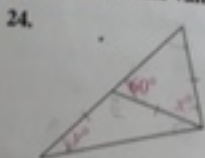
- a. Highlight an obtuse isosceles triangle in the design. Then find its angle measures.  
 b. How many different sizes of angles can you find in the logo? What are their measures?  
 c. **Open-Ended** Design a logo using isosceles triangles. Give the measures of the angles in your logo.

20. **Architecture** Seventeen spires, pictured at the left, grace the majestic Cadet Chapel at the Air Force Academy in Colorado Springs, Colorado. Each spire is an isosceles triangle with a  $40^\circ$  vertex angle. Find the measure of each base angle.

**Mental Math** Find the value of  $x$ .



**Algebra** Find the values of  $x$  and  $y$ .



nt Triangles

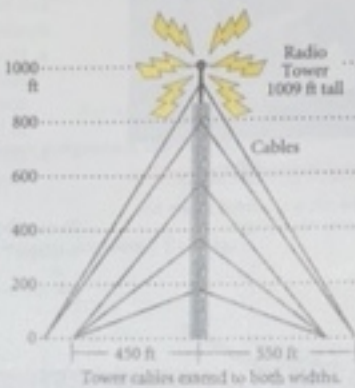
27. Write the Isosceles Triangle Theorem and its converse as a biconditional.
28. **Critical Thinking** An exterior angle of an isosceles triangle has measure 100. Find two possible sets of measures for the angles of the triangle.



**Connection**

casters  
ions  
ners.

29. **a. Communications** In the diagram at the right, what type of triangles are formed by the cables of the same height and the ground?
- b.** What are the two different base lengths of the triangles?
- c.** How is the tower related to each of the triangles?

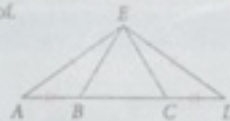
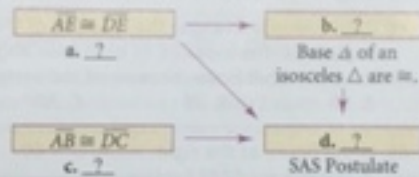


30. **Critical Thinking** Curtis defines the base of an isosceles triangle as its "bottom side." Is his definition a good one? Explain.
31. **Reasoning** What are the measures of the base angles of an isosceles right triangle? Explain.
32. **Writing** Explain how each corollary on page 212 follows from its theorem. First, write one explanation and then write the second similar to the first.

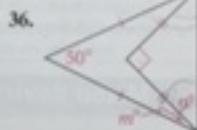
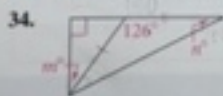
33. **Developing Proof** Copy and complete the flow proof.

Given:  $\overline{AE} = \overline{DE}$ ,  $\overline{AB} = \overline{DC}$

Prove:  $\triangle ABE \cong \triangle DCE$



34. **Algebra** Find the values of  $m$  and  $n$ .



**Challenge**

**Coordinate Geometry** For each pair of points, there are six points that could be the third vertex of an isosceles right triangle. Find the coordinates of each point.

37. (4, 0) and (0, 4)      38. (0, 0) and (5, 5)      39. (2, 3) and (5, 6)

40. **Algebra** A triangle has angle measures  $x + 15$ ,  $3x - 35$ , and  $4x$ .
- a.** Find the value of  $x$ .      **b.** Find the measure of each angle.
- c.** What type of triangle is it? Why?

**Proof** 41. Write a paragraph proof of Theorem 4-5 using the diagram next to it on page 211.

42. State the converse of Theorem 4-5. If the converse is true, write a paragraph proof. If the converse is false, give a counterexample.