CISES

and Problem Solving

Each conditional statement below is true. Write its converse. If the converse is also true, combine the statements as a biconditional.

1. If two segments have the same length, then they are congruent.

2. Algebra If $x = 12$, then $2x - 5 = 19$.

3. If a number is divisible by 20, then it is even.

4. Algebra If $x = 3$, then $|x| = 3$.

5. In the United States, if it is July 4th, then it is Independence Day.

6. Algebra If $x = -10$, then $x^2 = 100$.

Write the two statements that form each biconditional.

7. A line bisects a segment if and only if the line intersects the segment only at its midpoint.

8. An integer is divisible by 100 if and only if its last two digits are zeros.

9. You live in Washington, D. C., if and only if you live in the capital of the United States.

10. Two lines are parallel if and only if they are coplanar and do not intersect.

11. Two angles are congruent if and only if they have the same measure.

12. Algebra $x^2 = 144$ if and only if $x = 12$ or $x = -12$.

Test each statement below to see if it is reversible. If so, write it as a true biconditional. If not, write *not reversible.*

13. A perpendicular bisector of a segment is a line, segment, or ray that is perpendicular to a segment at its midpoint.

14. Parallel planes are planes that do not intersect.

15. A Tarheel is a person who was born in North Carolina.

16. A rectangle is a four-sided figure with at least one right angle.

17. A midpoint of a segment is a point that divides a segment into two congruent segments.

Is each statement below a good definition? If not, explain.

18. A cat is an animal with whiskers.

19. A dog is a good pet.

20. A segment is part of a line.

21. Parallel lines do not intersect.

22. A square is a figure with two pairs of parallel sides.

23. An angle bisector is a ray that divides an angle into two congruent angles.

24. **Language Arts** Is the following a good definition? Explain. An obtuse angle is an angle whose measure is greater than 90.

25. **Open-Ended** Choose a definition from a dictionary or from a glossary. Explain what makes the statement a good definition.

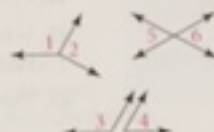26. **Writing** Write a definition of *a line parallel to a plane.*

Need Help?

For Exercise 27, complete this sentence: Two angles are a linear pair if and only if. . .

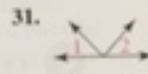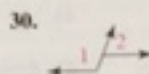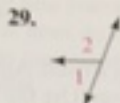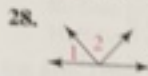**27. Writing** Use the figures below to write a good definition of *linear pair*.
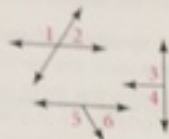
Linear pairs   Not linear pairs

Do angles 1 and 2 form a linear pair? Explain. (*Hint:* See Exercise 27.)

28.   **29.**   **30.**   **31.**

x^2 **Algebra** Each conditional statement is true. If the converse is true, write a biconditional. If not, provide a counterexample to show that the converse is false.
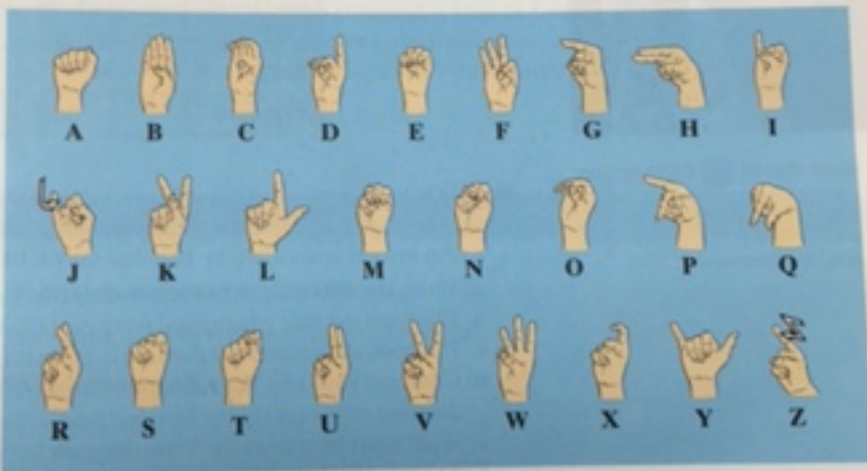
32. If $x = 19$, then $2x - 3 = 35$.   **33.** If $x = 3$, then $x^2 = 9$.

34. If $x > 0$, then $|x| > 0$.   **35.** If $x = 5$, then $x^3 = 125$.

The American Manual Alphabet For Exercises 36–40, use the chart below. Decide whether the description of each letter is a good definition. If not, provide a counterexample by giving another letter that could fit the definition.

36. The letter D is formed by pointing straight up with the finger beside the thumb and folding the other fingers and the thumb so that they all touch.

37. The letter K is formed by making a V with the two fingers beside the thumb.

38. You have formed the letter Y if and only if the thumb and one finger are pointing up and the other fingers are folded into the palm of your hand.

39. You have formed the letter I if and only if the smallest finger is sticking up and the other fingers are folded into the palm of your hand with your thumb folded over them, and your hand is held still.

40. You form the letter B by holding all four fingers tightly together and pointing them straight up while your thumb is folded into the palm of your hand.

Real-World Connection

The five letters above form a word to think about.

Write each statement as a biconditional.

41. Congruent angles are angles with equal measure.

42. When the sum of the digits of an integer is divisible by 9, the integer is divisible by 9 and vice versa.

43. The whole numbers are the nonnegative integers.

Reading Math Let p be the statement "$\angle A$ is an acute angle." Let q be the statement "$\angle A$ has measure between 0 and 90." Substitute for p and q and write each statement the way you would read it.

44. $p \rightarrow q$

45. $q \rightarrow p$

46. $p \leftrightarrow q$

47. Reasoning In a band, Amy, Bob, and Carla are the drummer, guitarist, and keyboard player. Use the clues to find the instrument that each one plays.

Carla and the drummer wear different-colored shirts.
The keyboard player is older than Bob.
Amy, the youngest band member, lives next door to the guitarist.

You can solve this type of logic puzzle by eliminating possibilities. Copy the grid below. Put an X in a box once you eliminate it as a possibility.

Instrument	Amy	Bob	Carla
Drums			
Guitar			
Keyboard			
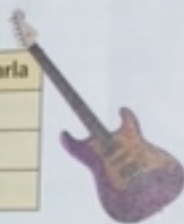
48. You have illustrated true conditional statements with Venn diagrams. You can do the same thing with true biconditionals. Consider the following statement.
An integer is divisible by 10 if and only if its last digit is 0.
a. Write the two conditional statements that make up this biconditional.
b. Illustrate the first conditional from part (a) with a Venn diagram.
c. Illustrate the second conditional from part (a) with a Venn diagram.
d. Combine your two Venn diagrams from parts (b) and (c) to form a Venn diagram representing the biconditional statement.
e. What must be true of the Venn diagram for any true biconditional statement?
f. Reasoning How does your conclusion in part (e) help to explain why a good definition can be written as a biconditional?

49. Reasoning Alan, Ben, and Cal are seated as shown with their eyes closed. Diane places a hat on each of their heads from a box they know contains 3 red and 2 blue hats. They open their eyes and look forward.

Alan says,
"I cannot deduce what color hat I'm wearing."
Hearing that, Ben says, "I cannot deduce what color I'm wearing, either."
Cal then says, "I know what color I'm wearing!"

How does Cal know the color of his hat?